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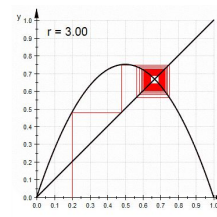
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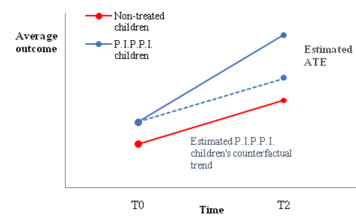
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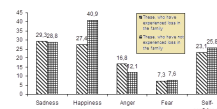
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Discrete Analogue of the Verhulst Equation and Attractors. Methodological Aspects of Teaching

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Abstract. Verhulst equation in differential and discrete form is very important in different fields of material science, sociology and economics. Methodological aspects of teaching, when both models are presented for solving advanced tasks, are considered. Solutions presented in graphical and numerical forms are discussed in the framework of attractors.

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Keywords: Logistic Equation; Verhulst Equation; attractors; chaos; teaching.

Short title: Logistic Equation: Teaching.

Introduction

Logistics is the art of computing. In the middle of the XIX century, Belgian mathematician Pierre François Verhulst studied the population growth. He established that initial stage of growth is approximately exponential; then, as saturation begins, the growth slows, and at the maturity, growth stops. In 1838 Verhulst introduced the logistic equation with a maximum value for the population (partial logistic growth model) [1]. A typical application of the logistic equation is a *common model of population growth*.

Applications of logistics function are useful in many fields, including material science (chemistry, geoscience), bioscience (biomathematics, artificial neural networks, ecology), sociology (political science, mathematical psychology, demography), economics (spreading of innovations, finance), linguistics (machine learning).

Presented topic is included in the master study course of *Applied mathematical methods* in the study programme *Information systems* (ISMA, Riga, Latvia).

This work is devoted to estimation of the methodological aspects of teaching, when differential and discrete models for solving the advanced tasks are presented. Moreover, the profit of attractors is discussed in student-friendly manner.

1. Literature review

Traditional estimation of dynamical systems is described by Robinson [2]. Several useful mathematical methods based on iterations allow estimating the one-dimensional dynamics and describing the chaos as a determined system. Also, Strogatz [3] represents an overview of mentioned systems for practical needs in natural sciences. Pearl [4] analyses the cause and effect relations which are fundamentally deterministic. He pointed out that cause and effect analysis must be

estimated using probability factor.

Generally, conception of chaos was presented and analysed by Holmgren [5], Kinnunen [6], Alligood [7]. Peitgen [8] describes several types of attractors. Muray [9] analyses the biological oscillators. Kapica et al. [10] represent the complicated structures with bifurcational behaviour.

2. Logistic growth model

The main idea was formulated by Verhulst [1]: the rate of reproduction is proportional to both the existing population and the amount of available resources (all else being equal).

Let $x=x(t)$ represents the population size at time t , when k is the maximum possible population size (the capacity of the environment), $x \in [0; k]$. Two initial assumptions for deriving the equation are presented below.

1. The rate of reproduction of the population is proportional to its current value x .
2. The rate of reproduction of the population is proportional to the amount of available resources which, in turn, is proportional to the value $(k-x)$:

$$k - x = k \left(1 - \frac{x}{k} \right). \quad (1)$$

Note that fight for resources limits the growth of the population.

The rate of reproduction is the derivative of x with respect to t . The equation can be represented in the form

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{k} \right), \quad (2)$$

where parameter $r > 0$ represents the coefficient of proportionality characterizing the rate of population growth. We will consider different values of r . The initial population size is given by the initial condition: $x(0)=x_0$.

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Solution of equation. Note that the functions $x=0$ and $x=k$ for all t are solutions of this equation, so we solve the equation for $x \in (0; k)$. Making the substitutions

$$y = \frac{x}{k}, \quad x = ky, \quad \frac{dx}{dt} = k \frac{dy}{dt}, \quad (3)$$

where

$$y = y(t), \quad y(0) = y_0, \quad y_0 = \frac{x_0}{k}, \quad (4)$$

we get the following equations for $y \in (0; 1)$:

$$\frac{dy}{dt} = ry(1 - y) \quad (5)$$

or in differential form:

$$dy = ry(1 - y)dt. \quad (6)$$

Eq.(6) represents the equation with separable variables. After separation

$$\frac{dy}{y(1 - y)} = rdt, \quad (y \neq 0, \quad y \neq 1), \quad (7)$$

we integrate it in timescale from 0 to t :

$$\int_{y_0}^y \frac{dy}{y(1 - y)} = \int_0^t rdt. \quad (8)$$

We represent the integrand in the form

$$\int_{y_0}^y \left(\frac{1}{y} - \frac{1}{y - 1} \right) dy = rt \quad (9)$$

and we get

$$(\ln |y| - \ln |y - 1|) \Big|_{y_0}^y = rt. \quad (10)$$

According to Newton-Leibniz formula,

$$\ln \left| \frac{y}{y - 1} \right| - \ln \left| \frac{y_0}{y_0 - 1} \right| = rt. \quad (11)$$

As $y \in (0; 1)$, then

$$\left| \frac{y}{y - 1} \right| = \frac{y}{1 - y}. \quad (12)$$

Considering this condition and the fact, that the difference of the logarithms is equal to the logarithm of the fraction, we get:

$$\ln \frac{y(1 - y_0)}{(1 - y)y_0} = rt, \quad \frac{y(1 - y_0)}{(1 - y)y_0} = e^{rt}, \quad (13)$$

$$\frac{1 - y}{y} = \frac{1 - y_0}{y_0 e^{rt}}, \quad \frac{1}{y} - 1 = \frac{1 - y_0}{y_0 e^{rt}}, \quad (14)$$

$$\frac{1}{y} = \frac{1 - y_0 + y_0 e^{rt}}{y_0 e^{rt}}, \quad (15)$$

$$y = \frac{y_0 e^{rt}}{1 - y_0 + y_0 e^{rt}}. \quad (16)$$

Consider the behaviour of the solution Eq.(16) at infinity:

WolframAlpha computational intelligence.

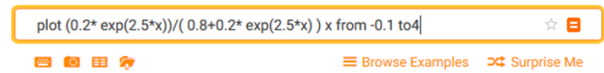


Fig. 1. Logistic function presented in Wolfram Alpha style.

$$\lim_{t \rightarrow +\infty} y(t) = \lim_{t \rightarrow +\infty} \left(\frac{y_0 e^{rt}}{1 + y_0(e^{rt} - 1)} \right), \quad (17)$$

$$\lim_{t \rightarrow +\infty} y(t) = \lim_{t \rightarrow +\infty} \left(\frac{y_0}{\frac{1}{e^{rt}} + y_0(1 - \frac{1}{e^{rt}})} \right) = \frac{y_0}{y_0} = 1. \quad (18)$$

Using that substitutes

$$y = \frac{x}{k}, \quad y_0 = \frac{x_0}{k} \quad (19)$$

in Eq.(16), we get

$$\frac{x}{k} = \frac{\frac{x_0}{k} e^{rt}}{1 - \frac{x_0}{k} + \frac{x_0}{k} e^{rt}} \quad (20)$$

and, consequently, an exact solution of Eq.(2) is so called *logistic function*

$$x(t) = \frac{kx_0 e^{rt}}{k + x_0(e^{rt} - 1)}, \quad (21)$$

where x_0 represents the initial size of population. For function $x(t) = ky(t)$, according to Eq.(18),

$$\lim_{t \rightarrow +\infty} y(t) = 1, \quad \lim_{t \rightarrow +\infty} x(t) = k, \quad (22)$$

here k represents the capacity of the environment, as the maximum possible size of the population. This solution does not give periodic solutions or any deviations.

For drawing of the logistic function, comprehensive list of mathematical software includes *Mathcad* [11] and *Wolfram* [12]. For example, Fig. 1 represents the expression in *Wolfram* style. Fig. 2 represents the plot of logistic function expressed by Eq.(21).

Discrete analogue of the Verhulst equation. We consider the Verhulst equation

$$\frac{dy}{dt} = ry(1 - y). \quad (23)$$

Let's assume the discrete time scale: $t=0, 1, 2, \dots$ (time changes discretely). We denote by

$$y(0) = y_0, \quad y(1) = y_1, \quad y(2) = y_2, \quad \dots \quad (24)$$

Generally,

$$y(t) = y_t, \quad y(t + 1) = y_{t+1}, \quad (25)$$

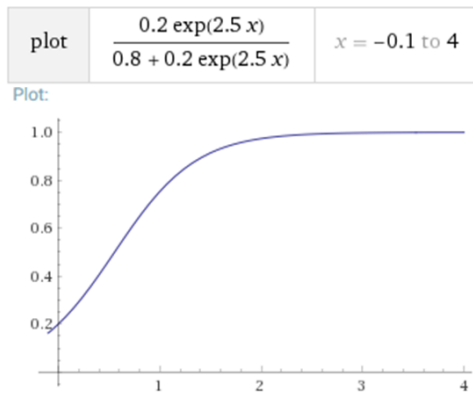


Fig. 2. Graph of logistic function expressed by Eq.(21): $r=2.5; k=1.0; x_0=0.2$.

where y_t represents the population size at year t . Since the derivative of the function is the limit of the ratio of the function increment to the argument increment, we can assume that the derivative is approximately equal to presented ratio:

$$\frac{dy}{dt} \approx \frac{\Delta y}{\Delta t}. \tag{26}$$

In our case

$$\Delta t = (t + 1) - t = 1, \quad \Delta y = y_{t+1} - y_t. \tag{27}$$

For Verhulst equation Eq.(23), taking into account the approximation expressed by Eq.(26), we receive the equation

$$\frac{y_{t+1} - y_t}{1} = r y_t (1 - y_t), \tag{28}$$

$$y_{t+1} = (1 + r) y_t - r y_t^2. \tag{29}$$

By transforming to another form

$$y_{t+1} = (1 + r) y_t \left(1 - \frac{r}{1 + r} y_t \right) \tag{30}$$

and after replacement

$$\frac{r}{1 + r} y_t = x_t, \quad y_t = \frac{1 + r}{r} x_t, \tag{31}$$

as a result, we obtain the new one:

$$x_{t+1} = (1 + r) x_t (1 - x_t). \tag{32}$$

Using r instead of $(1+r)$, we obtain a discrete analogue of the Verhulst equation:

$$x_{t+1} = r x_t (1 - x_t). \tag{33}$$

We will study the properties of Eq.(33) at different values of the parameter r . Two-dimensional graphs presented in Figs. 3-4, 6-11 were prepared using *Cobweb* software [13].

Let's consider the construction path of the sequence (x_t) . We use two functions $y=x$ and $y=r x(1-x)$ presented in Fig. 3. The vertex of a parabola is a point with coordinates $(0.5; r/4)$, $x=0$ and $x=1$ are zeros of the quadratic function.

First step. We take an arbitrary initial condition x_0 . We draw a vertical line to the intersection with the parabola. On the y-axis we obtain the value x_1 . Now we use a straight line $y=x$, we transfer the value x_1 to the x-axis.

Second step. At the point x_1 , let's draw a vertical line to the intersection with the parabola. On the y-axis we obtain the value x_2 . Now we use a straight line $y=x$, we transfer the value x_2 to the x-axis.

Third and following steps. This step will be realized using the same routine. Fig. 3 represents geometrical view of the sequence formation: $x_0, x_1, x_2, x_3, \dots$, using the functions $y=x$ and $y=r x(1-x)$, $r=1.78$.

If you do not follow the same path twice in the forward and backward directions, you may limit yourself to a broken line: a vertical line from the initial value x_0 to the intersection with the parabola, then the horizontal line to the intersection with the straight line $y=x$. Let's keep doing it again and again, the vertical line to the intersection with the parabola and the horizontal line to the intersection with the straight line $y=x$, etc, as shown in Fig. 4.

Let's consider the behaviour of the sequence (x_t) at different values of parameter r . For $0 \leq r < 3$, three different characteristic types of behaviour could be established.

Let's consider Eq.(33) with parameters $r=0.5$, $r=1.6$, $r=2.87$ as examples for analysing. In case when $r=0.50$ (see Fig. 5), the sequence (x_t) converges to zero for any initial value x_0 . In case when $r=1.60$ (see Fig. 6), the sequence (x_t)

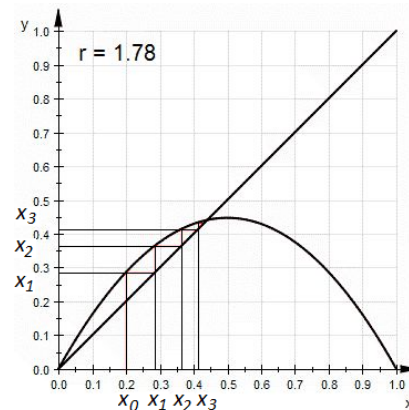


Fig. 3. Formation of sequence $x_0, x_1, x_2, x_3, \dots$, using the functions $y=x$ and $y=r x(1-x)$, $r=1.78, x_0=0.2$.

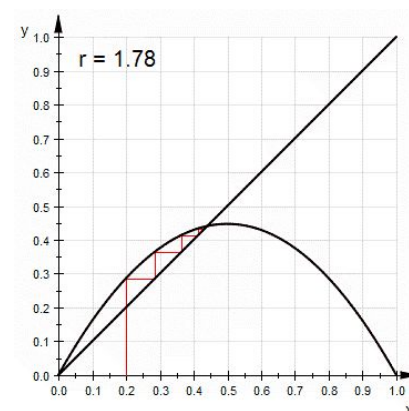


Fig. 4. Schematic representation of the sequence at $r=1.78, x_0=0.2$ (according to Fig. 3).

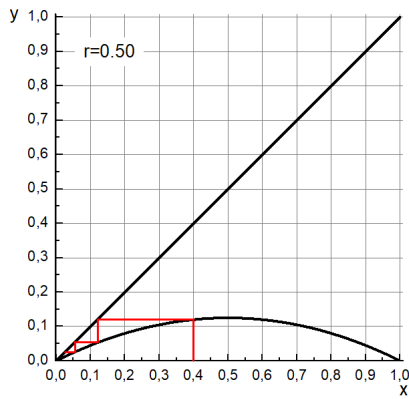


Fig. 5. Sequence (x_t) converges to $x^*=0$ for $x_0=0.4$ at $r=0.50$.

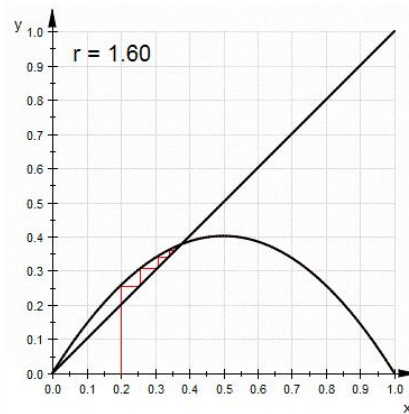


Fig. 6. Sequence (x_t) converges to $x^*=0.375$ for $x_0=0.2$ at $r=1.60$.

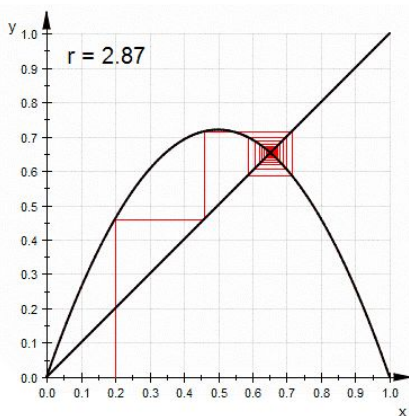


Fig. 7. Sequence (x_t) converges to $x^* \approx 0.65$ for $x_0=0.2$ at $r=2.87$.

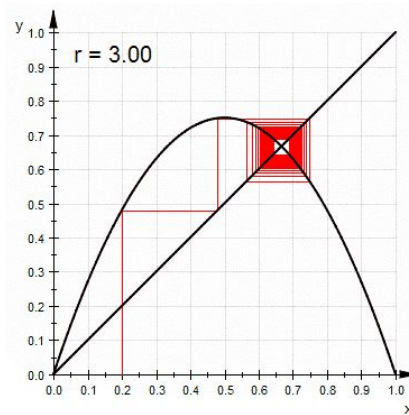


Fig. 8. Sequence (x_t) behaviour near the point $x^*=2/3$ at $r=3.00$.

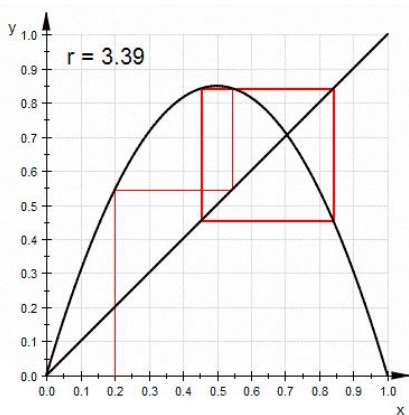


Fig. 9. Sequence (x_t) . Oscillation between two values $x \approx 0.84$ and $x \approx 0.46$ at $r=3.39$.

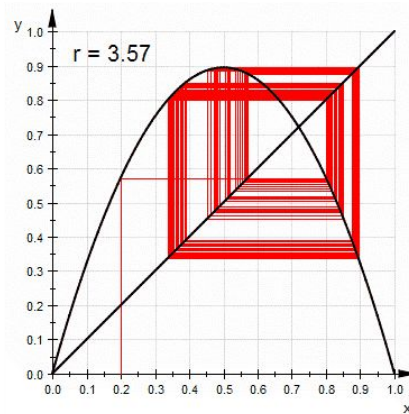


Fig. 10. Sequence (x_t) oscillates in chaotic manner at $r=3.57$.

converges to $x^*=0.375$ quickly enough for any value x_0 . In case when $r=2.87$ (see Fig. 7), the sequence (x_t) converges to the value $x^* \approx 0.65$ quite slowly - not as fast as in case when $r=1.60$.

For $3 \leq r \leq 4$, the behaviour of sequence (x_t) changes. Let's consider $r=3.00$, $r=3.39$, $r=3.57$, $r=3.93$ as examples for analysing. When $r=3.00$ (see Fig. 8), the sequence (x_t) approaches to the point $x^*=2/3$ very slowly. When $r=3.39$ (see

Fig. 9), the sequence (x_t) oscillates between two values $x^* \approx 0.84$ and $x^* \approx 0.46$.

When $r=3.57$, convergence and oscillations are absent, and values of sequence (x_t) are distributed in chaotic manner with several groups of periodic frames - see Fig. 10. With increasing of r up to value 3.93, the same behaviour will be kept, and number of periodic frames increases - see Fig. 11.

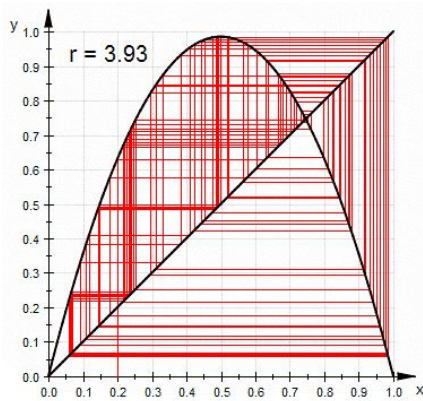


Fig. 11. Sequence (x_t) is distributed chaotic for any x_0 at $r=3.93$.

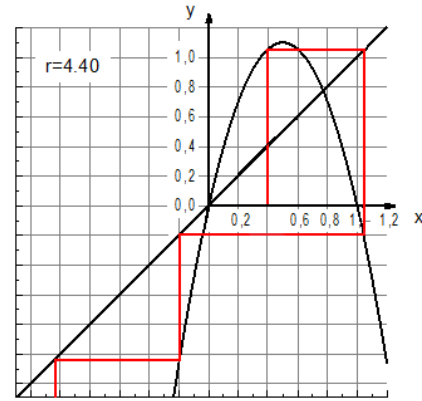


Fig. 12. Sequence (x_t) for any x_0 tends to $-\infty$ at $r=4.40$.

3. Attractors. Conditions for stability

Let's assume the transition of current system from one state to another. Let the transition be described by the equation

$$x_{t+1} = F(x_t) = rx_t(1 - x_t) \quad (34)$$

and the initial conditions of the system are chosen arbitrarily, then the final behaviour of the system is described by a point or a set of points.

A point or set of points that attract all nearest points to it is called an attractor. We will consider three kinds of attractors:

- i) a fixed-point attractor;
- ii) a limit cycle attractor or a periodic attractor;
- iii) chaotic attractor or strange attractor.

According to the kind of attractors, the behaviour of a non-linear system could be classified into following groups:

- a) stable and converging to an equilibrium value;
- b) oscillating in a stable limit cycle;
- c) chaotic, but bounded;
- d) unstable and exploding.

Using the equation Eq.(34) when $x_t \in [0;1]$, r must be treated as a variable parameter. We will consider $0 \leq r \leq 4$, since when $r > 4$, sequence (x_t) tends to $-\infty$. This is an unstable and unlimited behaviour of the system: see Fig. 12 for sequence at $r=4.40$.

3.1. The fixed point attractor

In mathematics, a *fixed point* of a function is an element of the function's domain that is mapped to itself by the function. Accordingly, x^* is a fixed point of the function $F(x)$ if $F(x^*)=x^*$. This means

$$F^2(x^*) = F(F(x^*)) = F(x^*) = x^*, \quad (35)$$

$$F^n(x^*) = x^*, \quad n \in N, \quad (36)$$

an important terminating consideration when recursively computing F .

Let's consider the equation

$$x = F(x). \quad (37)$$

Function F is a contracting map in a closed interval $I \in R$ if F meets two following conditions.

1. $F: I \rightarrow I$, where I is a closed interval. If $x \in I$, then $F(x) \in I$.
2. F is the contraction on this interval, if some $L \in (0;1)$ exists, such that inequality

$$|F(x) - F(x')| \leq L |x - x'| \quad (38)$$

is valid for any $x, x' \in I$.

Then, according to the principle of contracting map, the equation $x=F(x)$ has a unique solution $x^* \in I$, and for any initial condition $x_0 \in I$ sequence (x_t) , $t=0,1,2, \dots$, determined by the condition $x_{t+1}=F(x_t)$, converges to the value x^* .

Passing to the limit in Eq.(34), we get

$$x^* = rx^*(1 - x^*). \quad (39)$$

Therefore, we find the fixed point x^* by solving equation

$$x = F(x), \quad (40)$$

where

$$F(x) = rx(1 - x). \quad (41)$$

One solution of the equation

$$x = rx(1 - x). \quad (42)$$

is $x=0$. Then, if $x \neq 0$, $1=r \cdot (1-x)$, and another solution is

$$x = \frac{r - 1}{r}. \quad (43)$$

It means that two fixed points will be determined as follows:

$$x_1^* = 0; \quad x_2^* = \frac{r - 1}{r}. \quad (44)$$

If $r \in (0;1)$, then $r-1 < 0$ and x_2^* does not belong to the interval $[0;1]$.

Let's start to analyse the function F expressed from Eq.(41):

$$F(x) = rx - rx^2. \quad (45)$$

Table 2. Conditions of the stability at fixed point.

$ F'(x) _{x=x^*} < 1$	locally stable	attracting
$ F'(x) _{x=x^*} = 1$	neutral stable	non-attracting non-repulsive
$ F'(x) _{x=x^*} > 1$	unstable	repulsive

According to the condition that the point x^* is the limit of the sequence (x_t) , it is necessary that F meets the second condition - see Ineq.(38). For $F(x)$, the first condition fulfilled at interval $[0;1]$. This means that if $x \in [0;1]$, then $F(x) \in [0;1]$. Second condition will be satisfied, if

$$|F'(x)|_{x=x^*} < 1 \tag{46}$$

According to the condition $|F'(x)|_{x=x^*} < 1$, if the tangent of the slope of the function graph at the point x^* lies in interval $(-1;1)$, this fixed point is called as *locally stable*. If $|F'(x)|_{x=x^*} = 1$, the tangent to the graph of the function coincides with the line $y=x$ or $y=-x$. When $|F'(x)|_{x=x^*} = 1$, the point x^* is called as *neutral stable*: it ceases to be stable (attract sequence x_t), but not repulsive, i.e. is not unstable. In case if $F'(x)|_{x=x^*} = 0$, point x^* is called as super-stable.

Conditions of the stability at fixed point are presented in Table 2.

Let's express the first derivative of F :

$$F'(x) = r - 2rx. \tag{47}$$

We calculate $F'(x)$ at points x_1^* and x_2^* using Eqs.(44):

$$F'(0) = r, \tag{48}$$

$$F'\left(\frac{r-1}{r}\right) = r - 2r \cdot \frac{r-1}{r} = r - 2r + 2 = 2 - r. \tag{49}$$

Note that $|r|=r$ for $r \geq 0$ and

$$|2-r| = |r-2|. \tag{50}$$

Three possible cases for point x_2^* are presented below.

If $|r-2| < 1$, then $-1 < r-2 < 1$, it means $1 < r < 3$.

If $|r-2| = 1$, then $r-2 = -1$ or $r-2 = 1$, it means $r=1$ or $r=3$.

If $|r-2| > 1$, then $r-2 < -1$ or $r-2 > 1$, it means $r < 1$ or $r > 3$.

Table 3 represents the behaviour of the fixed points. Table 4 represents the dependence of the stability form at the fixed point on the value of the parameter r .

Table 3. Behaviour of the fixed points.

Fixed point	Locally stable (attracting) $ F'(x) _{x=x^*} < 1$	Neutral stable $ F'(x) _{x=x^*} = 1$	Unstable $ F'(x) _{x=x^*} > 1$
$x_1^*=0$	$0 \leq r < 1$	$r=1$	$r > 1$
$x_2^*=(r-1)/r$	$1 < r < 3$	$r=1$ or $r=3$	$r < 1$ or $r > 3$

Table 4. Dependence of the form of stability at a fixed point on the value of the parameter r

	$r \in (0;1)$	$r=1$	$r \in (1;3)$	$r=3$	$r \in (3;4]$
$x_1^*=0$	Locally stable	Neutral stable	Unstable	Unstable	Unstable
$x_2^*=(r-1)/r$	$x_2^* < 0$	$x_2^*=0=x_1^*$	Locally stable	Neutral stable	Unstable

3.2. Periodic attractor

Table 4 represents the dependence of the stability form at the certain fixed points. Point $x_2^*=(r-1)/r$ becomes unstable, when $r > 3$. According to that, the behaviour of the point x_2^* changes from attraction to repulsion when $r=3$. Fig. 9 represents the plot of two functions, $y=x$ and $y=rx(1-x)$, when $r=3.39$. Two crossing points are $x_1^*=0$ and $x_2^* \approx 0.7$. The sequence (x_t) oscillates between two other points: $x \approx 0.46$ and $x \approx 0.84$.

Instead of one stable point, two new ones appear. It means that after a certain number of iterations the system begins to oscillate from one of these points to the other. These points can be found from equation $x = F^2(x)$.

Periodic attractor for $x_{t+2} = F^2(x_t)$. We know that

$$x_{t+2} = F(x_{t+1}) = F(F(x_t)) = F^2(x_t), \tag{51}$$

where

$$F(x_t) = rx_t(1-x_t). \tag{52}$$

According to that,

$$F^2(x_t) = F(F(x_t)), \tag{53}$$

$$F^2(x_t) = F(rx_t(1-x_t)), \tag{54}$$

$$F^2(x_t) = r^2x_t(1-x_t)(1-rx_t(1-x_t)), \tag{55}$$

$$F^2(x_t) = r^2x_t(1-x_t)(1-rx_t+rx_t^2), \tag{56}$$

$$F^2(x_t) = r^2x_t(1-rx_t-x_t+2rx_t^2-rx_t^3). \tag{57}$$

We denote by $x^{*(2)}$ the fixed points of equation

$$x = F^2(x). \tag{58}$$

To find them, let's solve equation

$$x = r^2x(1-rx-x+2rx^2-rx^3). \tag{59}$$

Similar as in previous case,

$$x_1^{*(2)} = 0. \tag{60}$$

Table 5. Horner's scheme.

	r^3	$-2r^3$	$r^3 + r^2$	$1 - r^2$
		+	+	+
$\frac{r-1}{r}$	↓	$r^3 - r^2$	$-(r-1)(r^2+r)$	$r^2 - 1$
		=	=	=
	r^3	$-r^3 - r^2$	$r^2 + r$	0

We divide both sides of the Eq.(59) by $x \neq 0$.

$$1 = r^2(1 - rx - x + 2rx^2 - rx^3). \tag{61}$$

We transform it to the form of cubic equation:

$$r^3x^3 - 2r^3x^2 + r^2(r+1)x + 1 - r^2 = 0 \tag{62}$$

and use the Horner's scheme. It is known that

$$x_2^* = \frac{r-1}{r} \tag{63}$$

is the root of Eq.(62). We use Horner's scheme presented in Table 5. We can find the remaining two roots solving the equation

$$r^3x^2 - r^2(1+r)x + r(r+1) = 0. \tag{64}$$

As $r \neq 0$, we divide the equation by r^2 :

$$rx^2 - (1+r)x + \frac{r+1}{r} = 0. \tag{65}$$

Let's calculate the discriminant D :

$$D = (1+r)^2 - 4(r+1) = r^2 - 2r - 3 \tag{66}$$

and express the solution in form:

$$x_{3,4}^{*(2)} = \frac{1+r \pm \sqrt{r^2 - 2r - 3}}{2r}. \tag{67}$$

Depending on the sign of the discriminant, we need to consider three different cases.

First case. $D > 0$, if $r < -1$ or $r > 3$. In this case for $r > 3$ we get two additional solutions:

$$x_{3,4}^{*(2)} = \frac{1+r \pm \sqrt{r^2 - 2r - 3}}{2r}. \tag{68}$$

Second case. $D = 0$, if $r = -1$ or $r = 3$. At $r = 3$, we obtain a solution:

$$x_3^{*(2)} = x_4^{*(2)} = \frac{1+r}{2r} = \frac{1+3}{2 \cdot 3} = \frac{2}{3}, \tag{69}$$

Table 6. Dependence of the amount of limit points on the parameter r .

$0 < r < 3$, two points	$r = 3$, two points	$r > 3$, four points
$x_1^{*(2)} = 0$ $x_2^{*(2)} = \frac{r-1}{r}$	$x_1^{*(2)} = 0$ $x_2^{*(2)} = x_3^{*(2)} = x_4^{*(2)} = \frac{2}{3}$	$x_1^{*(2)} = 0$ $x_2^{*(2)} = \frac{r-1}{r}$ $x_{3,4}^{*(2)} = \frac{1+r \pm \sqrt{r^2 - 2r - 3}}{2r}$

which coincides with the solution

$$x_2^{*(2)} = \frac{r-1}{r} = \frac{2}{3}. \tag{70}$$

Third case. $D < 0$, if $-1 < r < 3$. There are no additional solutions.

We obtain the following dependence of the amount of limit points on the parameter r as presented in Table 6. To check which of these points are attractive, you need to count the first derivative of $F^2(x)$ on x at these points and make sure that the inequality Ineq.(71) is satisfied:

$$\left| \frac{dF^2(x)}{dx} \Big|_{x=x^*} \right| < 1. \tag{71}$$

Example. For $r = 3.4$ we obtain four fixed points:

$$x_1^{*(2)} = 0; \tag{72}$$

$$x_2^{*(2)} = \frac{3.4-1}{3.4} = \frac{24}{34} = \frac{12}{17} \approx 0.706; \tag{73}$$

$$x_{3,4}^{*(2)} = \frac{3.4+1 \pm \sqrt{3.4^2 - 2 \cdot 3.4 - 3}}{2 \cdot 3.4}; \tag{74}$$

$$x_3^{*(2)} = \frac{4.4 - \sqrt{1.76}}{6.8} \approx 0.452; \tag{75}$$

$$x_4^{*(2)} = \frac{4.4 + \sqrt{1.76}}{6.8} \approx 0.842. \tag{76}$$

For $F^2(x)$ (see Eq.(57)), let's calculate the first derivative with respect to x :

$$F^2(x) = r^2(x - rx^2 - x^2 + 2rx^3 - rx^4); \tag{77}$$

$$\frac{dF^2(x)}{dx} = r^2(1 - 2rx - 2x + 6rx^2 - 4rx^3). \tag{78}$$

Now we will establish the form of stability at fixed points. The point $x_1^{*(2)} = 0$ is an unstable fixed point because

$$\left. \frac{dF^2(x)}{dx} \right|_{x=x_1^{*(2)}} = r^2 = 3.4^2 > 1. \tag{79}$$

The point $x_2^{*(2)} \approx 0.706$ is an unstable fixed point because

$$\left. \frac{dF^2(x)}{dx} \right|_{x=x_2^{*(2)}} \approx 3.4^2(1 - 2 \cdot 0.706 - 2 \cdot 3.4 \cdot 0.706 + 6 \cdot 3.4 \cdot 0.706^2 - 4 \cdot 3.4 \cdot 0.706^3) \approx 1.96 > 1. \quad (80)$$

The point $x_3^{*(2)} \approx 0.452$ is locally stable fixed point because

$$\left. \frac{dF^2(x)}{dx} \right|_{x=x_3^{*(2)}} \approx 3.4^2(1 - 2 \cdot 0.452 - 2 \cdot 3.4 \cdot 0.452 + 6 \cdot 3.4 \cdot 0.452^2 - 4 \cdot 3.4 \cdot 0.452^3) \approx -0.759. \quad (81)$$

The point $x_4^{*(2)} \approx 0.842$ is locally stable fixed point because

$$\left. \frac{dF^2(x)}{dx} \right|_{x=x_4^{*(2)}} \approx 3.4^2(1 - 2 \cdot 0.842 - 2 \cdot 3.4 \cdot 0.842 + 6 \cdot 3.4 \cdot 0.842^2 - 4 \cdot 3.4 \cdot 0.842^3) \approx -0.754. \quad (82)$$

In case of $r=3.4$, existence of two stable points is established. Fig. 13 represents the plots of functions $y=x$ and $y=F^2(x)$ (see Eq.(57)). Tangent of the slope of the graph at points $x_3^{*(2)} \approx 0.452$ and $x_4^{*(2)} \approx 0.842$ lies in interval $(-1;0)$.

We consider the case, when $r=3.51$ - see Fig. 14. We can see that all four crossing points of the functions $y=x$ and $y=F^2(x)$ are unstable. For the value of the parameter $r=3.51$, points $x_3^{*(2)}$ and $x_4^{*(2)}$ cease to be stable and both points generate two new points (each). This phenomenon is called period doubling. Fig. 15 represents the functions $y=x$ and $y=F^4(x)$, where four new crossing points $x_5^{*(4)}$, $x_6^{*(4)}$, $x_7^{*(4)}$ and $x_8^{*(4)}$ are appearing.

When r increases further, four stable points cease to be stable and generate eight new points. The points at which the solution doubles are called as *bifurcation points*. The bifurcation process continues, generating 16, 32, 64, ... stable points. These points can be found, and their stability is determined in the same way as in previous case.

When a stable point ceases to be stable, it no longer attracts points. However, if the value of the unstable fixed point is given as the initial condition of the system, then the stable fixed points do not attract these points, the system remains at these points. The point $x=1$ is not fixed, so as $F(1)=0$, then the initial condition $x_0=1$ generates the sequence $x_1=0$, $x_2=0$, $x_3=0$, ..., $x_t=0$, ...

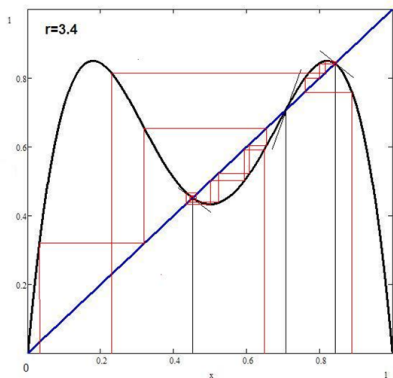


Fig. 13. $y=x$ and $y=F^2(x)$ at $r=3.4$. Four crossing points, but only two of them are stable for any x_0 .

Periodic attractor for $x_{t+4}=F^4(x_t)$. According to Eq.(55) we know that

$$F^2(x_t) = r^2 x_t(1 - x_t)(1 - r x_t(1 - x_t)). \quad (83)$$

Denote by

$$B(x_t) = 1 - r x_t(1 - x_t), \quad (84)$$

then

$$F^2(x_t) = r^2 x_t(1 - x_t)B(x_t). \quad (85)$$

Let's calculate F^3 :

$$F^3(x_t) = F(F^2(x_t)), \quad (86)$$

$$F^3(x_t) = r F^2(x_t)(1 - F^2(x_t)), \quad (87)$$

$$F^3(x_t) = r^3 x_t(1 - x_t)B(x_t)(1 - F^2(x_t)). \quad (88)$$

Let's denote by

$$C(x_t) = (1 - x_t)(1 - F^2(x_t)), \quad (89)$$

then

$$F^3(x_t) = r^3 x_t B(x_t) C(x_t). \quad (90)$$

Finally, let's calculate F^4 :

$$F^4(x_t) = F(F^3(x_t)), \quad (91)$$

$$F^4(x_t) = r F^3(x_t)(1 - F^3(x_t)), \quad (92)$$

$$F^4(x_t) = r^4 x_t B(x_t) C(x_t)(1 - r^3 x_t B(x_t) C(x_t)), \quad (93)$$

where $F^4(x_t)$ is the sixteenth-order polynomial with respect to x_t . We denote by $x^{*(4)}$ the fixed points of equation

$$x = F^4(x). \quad (94)$$

These solutions could be obtained by means of *Mathcad* software [11] using routine:

$$F^4(x) = x \quad \text{solve, } x \rightarrow \quad (95)$$

There are sixteen solutions of Eq.(94): eight solutions in real form and eight solutions in complex form as presented in Table 7. Only real solutions are considered here. Complex solutions are out of our interest. Among the set of solutions, $x_1^{*(4)}$, $x_2^{*(4)}$, $x_3^{*(4)}$, $x_4^{*(4)}$ correspond to the $x_1^{*(2)}$, $x_2^{*(2)}$, $x_3^{*(2)}$, $x_4^{*(2)}$ respectively as solutions of equation $x=F^2(x)$. Also, new solutions $x_5^{*(4)}$, $x_6^{*(4)}$, $x_7^{*(4)}$, $x_8^{*(4)}$ appear. Fig. 15 represents the plots of functions $y=x$ and $y=F^4(x)$ (Eq.(93)), where eight crossing points are indicated.

Table 7. Solutions of Eq.(94) at $r=3.51$ obtained using *Mathcad* [11].

Real solutions		Corresponds to the fixed point of F^2	Equation	Complex solutions	
n	$x_n^{*(4)}$			k	
8	0.87734182			1	0.98584709 - 0.00694482 i
4	0.85849118	$x_4^{*(4)} \rightarrow x_4^{*(2)}$	Eq.(67)	2	0.98584709 + 0.00694482 i
7	0.82501893			3	0.50562789 + 0.17578337 i
2	0.71509972	$x_2^{*(4)} \rightarrow x_2^{*(2)}$	Eq.(63)	4	0.50562789 - 0.17578337 i
6	0.50671306			5	0.16598412 + 0.07496759 i
3	0.42640911	$x_3^{*(4)} \rightarrow x_3^{*(2)}$	Eq.(67)	6	0.16598412 - 0.07496759 i
5	0.37772216			7	0.04914291 + 0.02368632 i
1	0	$x_1^{*(4)} \rightarrow x_1^{*(2)}$	Eq.(60)	8	0.04914291 - 0.02368632 i

3.3. Population behaviour and parameter r

If $r \in (0;1)$, the population will die out, regardless of the initial conditions.

$$x_1^* = 0. \tag{96}$$

If $r \in (1;2)$, the population size will quickly reach the stationary value

$$x_2^* = \frac{r-1}{r}, \tag{97}$$

regardless of the initial conditions.

If $r \in (2;3)$, the population size will also come to the same stationary value x_2^* , but will initially oscillate around it.

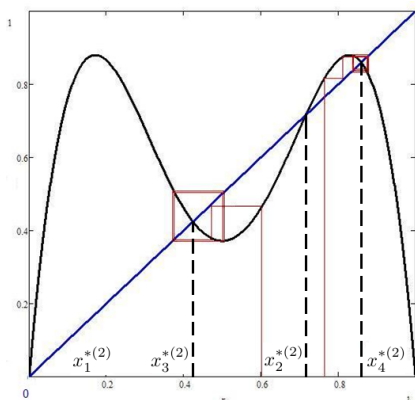


Fig. 14. $y=x$ and $y=F^2(x)$ at $r=3.51$. Four crossing points, all of them are unstable. Fixed points $x_n^{*(2)}$, $n=1,2,3,4$, are indicated by dashed line.

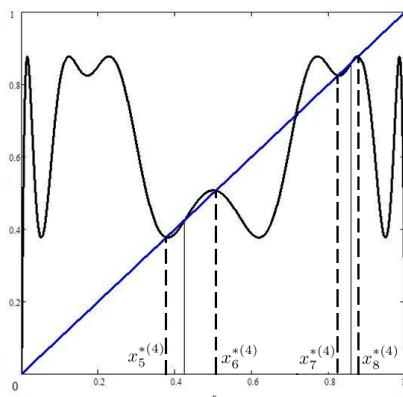


Fig. 15. $y=x$ and $y=F^4(x)$ at $r=3.51$. Eight crossing points. Fixed points $x_n^{*(4)}$, $n=5,6,7,8$, are indicated by dashed line.

If $r \in (3;1+\sqrt{6})$, where $1+\sqrt{6}=3.4495 \approx 3.45$, the population will fluctuate infinitely between two values [8],

$$x_{3,4}^{*(2)} = \frac{1+r \pm \sqrt{r^2-2r-3}}{2r} \tag{98}$$

and their value does not depend on x_0 .

If $r \in (1+\sqrt{6}; 3.54)$, then the population size will fluctuate between four values.

If $r > 3.54$, then the population size will fluctuate between 8 values, then 16, 32, etc.

Table 8 represents the dependence of fixed points x^* on r for bifurcational diagram. Bifurcation diagram demonstrates current attractor points for r values - see Fig. 16. The length of the interval at which the oscillations occur between the same number of values decreases as r increases. The ratio between the two system interval lengths tends to the first Feigenbaum constant $b=4.669201609\dots$ [3]. Such behavior is a typical example of a period doubling bifurcation cascade.

If $r \approx 3.57$, chaotic behaviour begins, and the doubling cascade ends. Fluctuations are no longer observed. Slight changes in the initial conditions lead to incomparable differ-

Table 8. Fixed point values for bifurcational diagram.

r	x_2^*
1.0	0
1.5	0.33
2.0	0.50
2.5	0.60
3.0	0.67

r	$x_3^{*(2)}$	$x_4^{*(2)}$
3.0	0.67	0.67
3.1	0.56	0.76
3.2	0.51	0.79
3.3	0.48	0.82
3.4	0.45	0.84
3.45	0.44	0.85

r	$x_5^{*(4)}$	$x_6^{*(4)}$	$x_7^{*(4)}$	$x_8^{*(4)}$
3.45	0.439	0.446	0.850	0.852
3.47	0.403	0.479	0.835	0.866
3.49	0.389	0.495	0.829	0.872
3.51	0.378	0.507	0.825	0.877
3.52	0.373	0.512	0.823	0.879
3.54	0.365	0.522	0.820	0.883

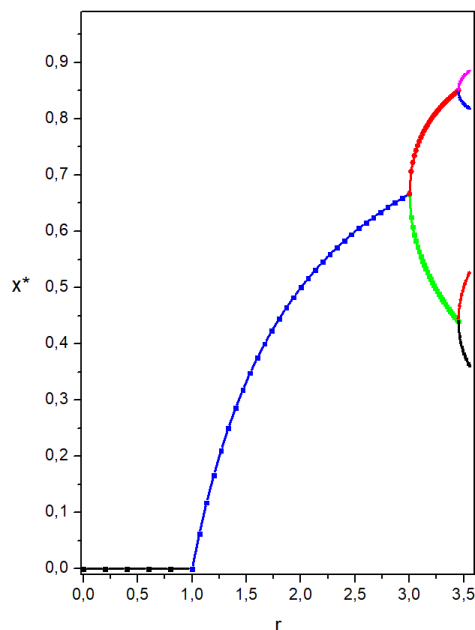


Fig. 16. Dependence of the attractor points x^* on parameter r . Bifurcation diagram.

ences in the future behaviour of the system in time, which is the main characteristic of chaotic behaviour.

For $r > 4$, the display values leave the interval $[0;1]$ and diverge under any initial conditions.

Conclusions

The discrete analogue of the Verhulst equation is interesting due to the following circumstance: for sequence Eq.(33) at different values of the parameter r , a set of different attractors (fixed point attractor, periodic attractor, chaotic attractor) could be obtained.

Methodologically, modelling tasks constructed using the Verhulst equation enable to understand the chaotic behaviour in real complicated forms of global complexity.

It is necessary to point out that chaotic behaviour of the model system corresponding to the real system depends on the method precision. Sensitivity of the model on initial conditions requires the detailed analysis of the stationary as well as dynamic behaviour.

References

1. N. Bacaer. A Short History of Mathematical Population Dynamics. – Springer-Verlag London Limited, 2011. – DOI 10.1007/978-0-85729-115-8-6.
2. C. Robinson. Dynamical systems. Stability, symbolic dynamics, and chaos. – CRC Press, 1995.
3. S.H.Strogatz. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry and engineering. – Westview Press, 2001.
4. Pearl J. Causality: Models, reasoning and inference. – Cambridge, UK: Cambridge University Press, 2000.
5. R.A.Holmgren. A first course in discrete dynamical systems. Sec. Edition, Springer-Verlag, 2000.
6. Tuula Kinnunen. On the way to the chaos. – Publications of the Turku school of Economics and Business Administration, Sarija/Series D-6, 1993, 134 p.
7. K. T. Alligood, T. D. Sauer, J. A. Yorke. Chaos. An introduction to dynamical systems. – Springer-Verlag, 1997.
8. Heinz-Otto Peitgen, Hartmut Jurgens, Dietmar Saupe. Chaos and Fractals. New Frontiers of Science. – Springer-Verlag. 1992.
9. J.D. Murray. Lectures on Nonlinear-Differential-Equation Models in Biology. – Oxford: Clarendon Press, 1977.
10. Капица С., Курдюмов С., Малинецкий Г. Синергетика и прогнозы будущего. 2-ое изд. (in Rus.) - Москва: Эдиториал УРСС, 2001. - 288 с.
11. Mathcad <<https://www.ptc.com/en/products/mathcad>>, accessed 2017 06 20.
12. Wolfram <<https://www.wolframalpha.com>>, accessed 2017 06 20.
13. Cobweb plot <https://en.wikipedia.org/wiki/Cobweb_plot#/media/File:LogisticCobwebChaos.gif>, accessed 2017 10 02.

Impact Evaluation of the P.I.P.P.I. Programme: a First Application of Counterfactual Analysis

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Abstract. The research focuses on the Italian Programme of Intervention for Prevention of Institutionalization (P.I.P.P.I.), the implementation of an integrated inter-professional, institutional and service method of intervention, at national level, with families at risk of child neglect. To prove the effectiveness of the programme with the (quasi) experimental evidence of Impact Evaluation, a set of non-participant families under the care of standard services are compared to participants of the 4th edition of the programme. Since a non-random selection process intervenes in the professionals' choice of the families to be included in the intervention, specific statistical techniques have been applied to minimize the selection bias resulting by comparing participants with non-participants. Even if environmental conditions are difficult to change, statistically significant effects on children's total risk of out-of-home placement and developmental needs satisfaction are estimated. Also, the effects on parents' response to child's needs are positive and, in general, professionals' support to parents becomes less important after they have participated in P.I.P.P.I.

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Short title: Quasi-experimental impact evaluation.

Introduction

Since the goal of an intervention is to improve a certain targets' condition or behaviour, to obtain indications about programme effectiveness is of crucial importance. Impact evaluation consists of statistical methods designed to assess the effect of a programme on the outcome variables to which the programme is addressed. The evaluator's task is to determine whether the observed changes in beneficiaries' outcomes can be related to the intervention in a "causal" sense (counterfactual approach [1,2]).

In Italy, impact assessment studies are still not widespread for cultural and feasibility reasons. This gap is even bigger when specific areas of the social field are considered. The present study is one of the first attempt to apply the counterfactual approach in the evaluation of a programme involving social, educational and health services for children and their families: the Italian experience of an intervention contrasting child neglect - *the Programme of Intervention for Prevention of Institutionalization* (P.I.P.P.I.).

In Italy the research on the phenomenon of child neglect is only at an early stage: the empirical knowledge about inter-

ventions effectively capable to support vulnerable parenting and, more specifically, to prevent child out-of-home placement is not yet developed. The intervention currently implemented by territorial services to support vulnerable families and their children is fragmented and not systematically organized across the country. There is not an ordinary and a codified practice of work with families, which explains the difficulties above mentioned in conducting research studies.

With the P.I.P.P.I. programme, the Italian government has invested for the first time in the national history of social policies on a comprehensive programme involving services located all along the peninsula with continuity and uniformity. A total of about 6,000 professionals and 2,000 families in more than 200 territories have been overall engaged across editions, from 2011 until now. Therefore, evaluation plays a key role for policy makers and all stakeholders responsible for the programme to obtain indications for their decisions.

In addition to providing an assessment of the programme effectiveness, the statistical analysis here reported will allow to achieve a deeper understanding of the children's and families' characteristics correlated with child neglect.

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This work is organized as follows:

- a) description of the programme P.I.P.P.I. at national and individual family level (section 1.1) with a focus on data collection tools (section 1.2);
- b) explanation of the evaluation strategy and statistical methods (section 2);
- c) empirical results, separately with the aim of describing vulnerable families (section 3.1) and to achieve an estimation of the programme impact (section 3.2);
- d) final discussion presenting results in synthesis, caveats and work in progress.

1. P.I.P.P.I. programme

1.1 The main features

Run through a collaboration between the Italian Ministry of Labour and Social Affairs and the University of Padua, the P.I.P.P.I. programme aims at contrasting child neglect in families where children are at risk of out-of-home placement. In line with the European 2030 Agenda for Sustainable Development, the programme experiments actions intended to break the cycle of social disadvantage (REC 2013/112/UE) by promoting positive parenting (REC 2006/19/UE).

At national level, the aim of the programme is harmonizing practices and models of intervention addressed to families in situation of child neglect through training, documentation and systematic evaluation shared throughout the entire country. Child neglect is defined as a significant deficiency or a failure to respond to the needs of a child recognized as fundamental on the grounds of the current scientific knowledge [3-4]. Considered as a complex social problem, in accordance with the bio-ecology of human development [5-6], responding to children's needs requires a collective action, which is not only a matter of helping parents, but also of promoting the responsibility of public institutions and social networks to develop relationships for families in their communities. For this reason, for each family in P.I.P.P.I., the programme provides the simultaneous functioning of four "specific activities" involving different ecosystem levels:

- 1) home-care intervention,
- 2) parents' and children's groups,
- 3) natural family helpers,
- 4) cooperation between school, families, social and health services.

These actions are integrated in a shared care plan, co-constructed for and with each child and family, by a multidisciplinary team (MT) of professionals following a *Participative and Transformative Evaluation* approach (PTE) [7].

At the individual child's level, P.I.P.P.I. proceeds in four stages, over a period of about 18 months. The intervention starts at T0 with the *PreAssessment* (phase 1), in which the MT completes a questionnaire for a wide range of households to decide which one to invite to be included in the programme. Once the family signs its agreement to enter the

programme, a deeper assessment and a planning stage follow (phase 2). This second step implies the definition of the MT of professionals (teachers, psychologists, social and health workers, voluntary family helpers, etc.) who, together with the family and the child themselves, negotiate and realize the specific individual micro-plans of interventions.

To achieve the specific purposes of the micro-plans, the realization of the programme provides the four activities above mentioned (phase 3) throughout the duration of the intervention. At the end of the programme (T2), a follow-up assessment is repeated with the same instruments already used at T0 (phase 4). This *PostAssessment* re-evaluates the actual conditions of the family and the child and helps the MT to decide whether the family should continue with P.I.P.P.I. or may move back to the standard care of social welfare support, the more or less intense the interventions are.

1.2 Data collection instruments

To drive theories and methods into practice, P.I.P.P.I. provides several tools that are used to support the care process within the PTE perspective. The main tools used by professionals to give voice to children and families and foster their participation, but also to facilitate communication within the MT, are the multidimensional model of the Child's World (CW), *RPMonline* and the *Pre-PostAssessment*. The data collected through these instruments are then used to evaluate the outcomes of families and children at both individual and aggregate level.

RPMonline is a web tool designed by the University of Padua to achieve a shared Assessment (*Rilevazione*) and care Plan (*Progettazione*) and to Monitor (*Monitoraggio*) the effectiveness of the measures taken to support families and children [8]. It is based on the CW model (the so-called "Triangle"), which consists in the Italian adaptation of the British Assessment Framework [9-10], a triangular representation of the three fundamental dimensions in considering a child:

- 1) the child's development needs;
- 2) the parental responses to meet these needs;
- 3) the environmental factors which may influence the response to child's needs.

Each CW dimension is also divided into a number of sub-dimensions, for a total of 17 variables. This instrument has been designed to get all points of view in the team, included children and families as well; thus, it facilitates negotiation between the different perspectives about the plan. The corresponding information is updated regularly to assess progress. Professionals can indeed document, for each sub-dimension, the contents of the qualitative analysis from the points of view collected and negotiated in the team, and record the corresponding quantitative synthesis through a score defined on a Likert scale (from 1="serious problem" to 6="strength"). This forms the CW Questionnaire [11], which must be neces-

sarily completed at times T0 and T2.

The *PreAssessment* is a questionnaire used by professionals before the beginning of the implementation (T0) to choose the families to be included in the programme. The same instrument, but in the version of *PostAssessment*, is completed by professionals at the end of the intervention (T2). The *PreAssessment* is used as a guide for a shared decision-making following the path of the PTE: the MT compares and negotiates the different points of view of the members to come to an agreement about the opportunity for a family to access the programme. The questionnaire, built as a checklist, has been created specifically for the P.I.P.P.I. implementation and was inspired by the works of Braconnier and Humbreeck [12]. It consists of five following sections:

- 1) vulnerability conditions and social interventions accessed by the family and the children;
- 2) family history;
- 3) risk and protection factors associated to each side of the CW, defined as scores on a six-point Likert scale (from 1=few factors to 6=many factors);
- 4) quality of the family and social services' relationship;
- 5) overall risk evaluation (from 1=absence of risk to 6=high risk), to be completed considering the previous sections.

Finally, the rich Questionnaire on Socio-Demographic Information is completed at T0 and constantly updated in *RPMonline*.

2. Evaluation strategy

In the P.I.P.P.I. programme, the simple pre-post comparison of results indicates a significant improvement in all outcome variables considered on families and children from the beginning to the end of the programme [13]. The question which counterfactual analysis aims to answer is how much of this positive change can be attributed to the participation in the intervention: what would have happened to children and families in P.I.P.P.I. if they did not take part in the programme but have been instead followed by the child protection mainstream activities?

To answer this question, it is necessary to compare the outcomes observed for the participants in P.I.P.P.I. with the results that would have been observed, on the same subjects, in the absence of the programme ("counterfactual" situation [14]). Denoted by Y the outcome variable which the programme likely affects, impact evaluation aims to determine how much Y changes in mean on the "treated" group at T2 as a result of the participation in the programme, the so-called *Average Treatment Effect*:

$$ATE = E(Y_{T2}^1 - Y_{T2}^0 | D = 1), \quad (1)$$

where $D=\{0,1\}$ is the indicator of exposure to "treatment" and variables Y^1/Y^0 refer to outcomes respectively with (1)

/ without (0) the programme.

Since it is not possible to observe the targets in the alternative situation of no participation in the programme, the counterfactual outcome of the beneficiaries ($Y_{T2}^0|D=1$) must be derived from the same outcome observed in a convenient comparison group of subjects not exposed to the intervention, the "control group" ($Y_{T2}^0|D=0$). By doing so, however exits the risk of confronting two groups of subjects systematically different from each other independently on the intervention and therefore attributing to it differences that would have been anyway observed (difference on the right-hand side of Eq.(3):

$$\hat{ATE} = E(Y_{T2}^1|D = 1) - E(Y_{T2}^0|D = 0), \quad (2)$$

$$\hat{ATE} = ATE + E(Y_{T2}^0|D = 1) - E(Y_{T2}^0|D = 0). \quad (3)$$

This a priori difference, known in the literature as "selection bias" [15], derives from the selection (self-selection) process by which the treated have been chosen (they chose) to participate in the intervention. Only in the ideal situation where assignment is random, the selection bias is null by construction. Because of practical and organizational reasons, as well as ethical motivations, in the experimentation of P.I.P.P.I. it is not possible to randomly decide which vulnerable families to include in the programme.

However, in the fourth implementation of P.I.P.P.I., for some local areas involved in the programme, the same data collected on the target families were also available for a comparison sample of vulnerable families under the standard care of social services (control families). This paper shows the comparison of the target families with the control ones, appropriately cleared from the selection bias by means of specific statistical techniques.

To estimate ATE, a two-steps evaluation strategy is applied. The average outcome at the end of the programme in the counterfactual setting of no participation in the intervention is firstly approximated by what is observed in the control group "re-sampled" thorough the Matching statistical procedure (step 1). Such technique allows to make a comparison fixed the conditions and the characteristics which potentially determine the selection bias [16-17]. Then, the combined use of the *Difference-In-Differences* (DID) estimator (step 2) allows to further control for the residual differences still exiting in the starting conditions between the two groups [18-19].

The first step implies to pair each treated family to a subgroup of control families as closely as possible with respect to a certain set of pre-treatment characteristics, X. The evaluator therefore assumes that two families with similar characteristics, one of which treated and the other not, would have presented the same outcomes if they both did not participate in the programme (Selection on observables assumption [20]):

$$E(Y_{T2}^0|D = 1, X) = E(Y_{T2}^0|D = 0, X). \quad (4)$$

Since the practical difficulty to find matches when there are many variables to control for, matching has been applied by

means of the propensity score [21], i.e. the conditional probability of receiving the treatment given the X :

$$p(X) = Pr(D = 1|X). \quad (5)$$

Only the need to obtain an estimate of the propensity score conditionally on which all covariates have approximately identical value is important in estimating the propensity score (Balancing property [22]). In the present evaluation study, various matching techniques have been tested to find the appropriate comparison group for the families participating in P.I.P.P.I. (see Ref.[23] for a detailed review of the alternative matching methods).

Kernel and Stratification Matching turned out to be preferable to improve the balancing of observables and also to obtain a comparison sample representative of the entire treated group of families. While the Kernel Matching considers all the members of the control group for each treated unit, with a weight inversely proportional to the distance between the two observed propensity scores, Stratification Matching divides the support of the propensity score into blocks and within each block, where the propensity score can be assumed approximately constant, compares treated and non-treated subjects.

At the second step, for each pair of readjusted comparison groups by means of matching, the DID estimator has been applied. Conditionally on X , this estimation strategy assumes the same trend in the average outcomes when non-participation in the programme is experienced for treated and controls:

$$E(Y_{T_2}^0 - Y_{T_0}^0|D = 1, X) = E(Y_{T_2}^0 - Y_{T_0}^0|D = 0, X). \quad (6)$$

Therefore, DID estimates the impact of the programme by the difference between (matched) participants and non-participants in the pre-post difference in outcomes:

$$ATE = E_X[ATE|X], \quad (7)$$

where $ATE|X$ is equal to

$$E(Y_{T_2}^1 - Y_{T_0}^0|D = 1, X) - E(Y_{T_2}^0 - Y_{T_0}^0|D = 0, X). \quad (8)$$

Fig. 1 represents the chart explaining how the DID estimator works. In addition to the simple difference of the pre-post change in the average outcomes, the DID estimator has been computed via the regression of the pre-post differences on the indicator of intervention participation plus other control variables. Since Kernel Matching turns out to be a re-sampling, the final estimators can be calculated by weighting the units of the baseline groups.

In Stratification Matching, where the support of the propensity score is divided in blocks satisfying the balancing property, ATE is alternatively computed by the weighted mean of the conditional ATE estimates, averaged over the blocks using the relative proportion of treated units (Blocking with regression [24]). All estimation procedures have

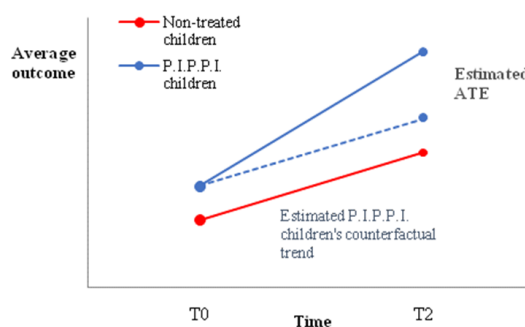


Fig. 1. The DID estimator.

been implemented using the statistical software *Stata*, which provides a specific package to perform propensity score matching [25].

3. Empirical results

Only 10 territories of the 46 overall involved in the fourth edition of the programme were selected to take part in the evaluation study, plus other two participating in the previous edition. The choice of the local areas to be included in the research was driven by a preliminary "territorial analysis" to test the starting conditions as well as the hypothesis of homogeneous treatment. To assure a good baseline comparison group for treated, practitioners were invited to follow the same inclusion criteria used for targets in the choice of the families to include in the control group. The same quantitative data, with the same tools and timelines, were collected in both the compared groups. The instruments were completed by professionals in a team also for the controls.

To facilitate the understanding of the instruments to operators who were not trained in the programme, professionals trained in P.I.P.P.I. (the "coaches") were present during the compilation with the task of supporting in the use of the tools and furthermore of conducting the assessment of the child and his/her family.

The number of families and children in each comparison group separately for local area is reported in Table 1: a total of 107 children in 97 families participating in P.I.P.P.I. vs. 146 children in 143 families in the care of standard services.

By directly comparing participants with non-participants, significant statistically differences are observed for some pre-treatment variables. This confirms the presence of selection bias, which prevents a direct comparison between the baseline groups to assess the programme effectiveness.

3.1. Propensity score and matching

To estimate the propensity score (i.e. the probability of entering the programme), a Logit model has been assumed. The specification of the model was driven by the need to obtain an estimate of the propensity score which satisfies the Balancing property and, at the same time, gives an accurate approximation to the conditional probability of participation in the pro-

Table 1. Families and children participating in the evaluation study.

Region	Territory	Treated		Non-treated	
		Families	Children	Families	Children
Emilia Romagna	Correggio-Guastalla	9	9	-	-
	Ferrara	8	8	-	-
	Modena	-	-	14	14
	Reggio Emilia	-	-	15	15
Lombardia	Bergamo	10	12	13	13
	Mantova	9	10	15	15
	Milano	9	10	15	15
Piemonte	Alessandria	9	11	14	14
	Fossano	9	9	15	15
	Torino	18	22	14	17
Veneto	Venezia	6	6	15	15
	Vicenza	10	10	13	13
Total		97	107	143	146

gramme.

Table 2 reports the results obtained from the estimation of the Logit model, where the average marginal effects on the probability to enter the programme are reported. These preliminary results of the counterfactual analysis allow to identify the key variables which come into play in the selection process of target families; it is in fact possible to outline more clearly the characteristics of the vulnerable families chosen by professionals to work with P.I.P.P.I., as compared to the vulnerable ones excluded [26].

The results, in general, refer to aspects which reduce the propensity score. It seems that vulnerable families owing to the presence of a traumatic and/or stressing event have been chosen to a greater extent. Risk conditions related to the perturbations of the family equilibrium because of conflict and absence of parents, stepfamily and adoption, as well as problems of disability or pathologies of parents are, instead, predictors of the non-participation in P.I.P.P.I.

Professionals tend to choose children in households with a background of transgenerational care; conversely, they seem to exclude those situations in which out-of-home placements has been experienced. The more parents have established a good relationship with professionals, the more families are likely to be involved in the intervention. Lastly, households whose vulnerability is the result of risky behaviours or conditions, such as alcohol or substance dependence, detention, degraded environment, etc., seem to be more frequently present among excluded families.

Fig. 2 shows the probability distribution of the estimated propensity score separately for the treated and the control group. On average, children in the treated group have a higher probability to be involved in the programme than the others.

In situations like this one, where the overlap condition is not fully satisfied, and the number of non-treated units is not much higher than the number of treated, Kernel and Stratification matching are recommended. By

re-sampling observations through matching, the comparison groups are much closer: none of the differences observed before matching is statistically significant and the problem of selection bias seems to be lessened.

Table 2. Marginal effects of the pre-treatment variables on the propensity score

Variable	Estimate
Child's and family information	
Age	0.9
Female	0.8
Foreigner family	-10.5
Number of children in the family	-2.6
Type of family	
Both biological parents	-17.8
Single parent	-6.2
Single parent and other adults	-18.8
Other types	-
Previous out-of-home placements	-15.9 *
Years duration of the care process	-0.8
Bad relationship between family and social services	-11.1 **
Vulnerabilities	
Transgenerational care	19.1**
Perturbations of family equilibrium	-17.5**
Parents' disability or psychiatric disease	-17.2*
Traumatic and/or stressing event	14.7**
At risk behaviors, conditions	-10.7**
Child abuse or witness of violence	-10.2
Child's disability or psychiatric disease	-9.0
Child neglect	7.7
Social deprivation	7.5
Economic deprivation	-6.0
Parents' psychological disease	4.9
Child's psychological disease	0.0

** {*} Difference statistically significant at the 95% {90%} level of confidence.

Average percentage marginal effects.

Robust standard errors clustered by territorial area.

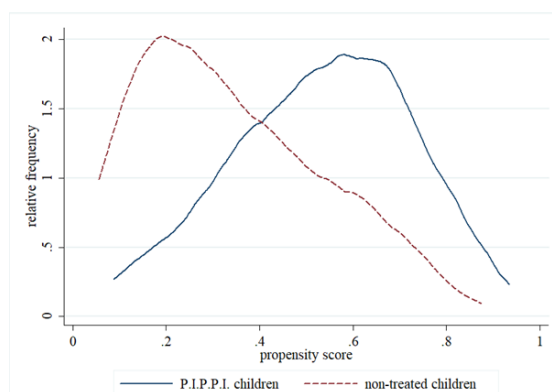


Fig. 2. Estimated probability distribution of the propensity score in the baseline treated and non-treated groups.

3.2. Estimated effects on outcomes

Table 3 reports the DID estimates of the ATE computed comparing the matched samples. The estimates refer to the average effect of the intervention in the group of the P.I.P.P.I. children on outcome variables measured at T2.

By "effect" we intend the difference at T2 between what is observed as a result of the intervention and what would have been observed in the counterfactual situation of care with standard services in mean on the P.I.P.P.I. child. The variables considered as outcomes are following.

1. The objective situation of the family at T2 in terms of greater parents' autonomy from professionals' support (dichotomous variable), defined "positive" (value=1) if the care process continues with lighter interventions or the family is no more in care because of an improvement in the situation.
2. The professionals' evaluation of the family situation at T2 (six-points Likert scores): all the sub-dimensions of the CW (17 variables); the number of protection and risk factors associated to each side of the CW – children's needs, parenting competences and environmental context (6 variables); an overall evaluation of the child's risk to be placed out-of-home.

Estimates are reported separately for type of matching and way of computation of the DID estimator. Type A estimations calculate the simple difference in the average pre-post changes; type B effects are derived from the estimation of a regression model of the pre-post differences on the treatment indicator plus other variables on which the differences in trends may depend: weighed regression for Kernel Matching and the simultaneous estimation of the conditional regressions within the blocks of the propensity score for Stratification Matching.

The results of the counterfactual analysis confirm the effectiveness of the intervention on most of the outcome variables considered. In particular, all methods agree in estimating a statistically significant impact on children's total risk

of out-of-home placement (the overall risk evaluation of the *PostAssessment*) and the satisfaction of the child's developmental needs (the child side of the "Triangle"), as well as on parents' achievement of a certain degree of "autonomy" in their work with services (the actual situation of the family at T2). While results are robust for the variables regarding children and environment, the effects estimated on the parental outcomes are statistically weaker and not consistent with the estimation method applied.

By summarizing the empirical evidence, we observed the following statements.

1. All methods show a significant effect on the total risk evaluation: the overall child's risk of out-of-home placement decreases by 0.47-0.55 points on the Likert scale 1-6 thanks to the participation in the programme. In other words, if the P.I.P.P.I. child had not entered the programme, but had been instead followed with the ordinary practices of social services, he/she would have achieved an average score of total risk evaluation significantly higher at T2.

2. The effect estimated for the impact on the family's work with services at T2 is also robust and statistical significant with a high degree of confidence. On average, the situation of the family at T2 is more frequently satisfactory thanks to P.I.P.P.I.: the conclusion of the care process because the situation has improved or the continuation of the work with services characterized by lighter interventions are events significantly more likely after P.I.P.P.I.

3. Concerning the CW Questionnaire, the impact of the programme is particularly high on the side of the child's needs; only for the sub-dimension "Health & Physical Development" P.I.P.P.I. and the ordinary services do not differ significantly. Even with regard to risk and protection factors, those one related to the child's developmental needs appear to reduce and increase, respectively, as a result of the participation in the programme.

4. Only with respect to a few sub-dimensions of the family and the environmental dimensions of the CW, P.I.P.P.I. and the ordinary services differ significantly. On the family side, only the sub-dimensions "Play, encouragement & fun" and "Parents' self-realization" seem to be affected by the programme; on the environmental side, "Employment & income", "Housing" and "Relationship with school and other services" improve to a greater extent, thanks to P.I.P.P.I.

5. As far as environmental conditions are concerned, the correspondent levels of protection and risk factors recorded by professionals in the *Pre-PostAssessment* seem to be unaffected by the programme.

Conclusions

The simple pre-post comparison of the variables to which P.I.P.P.I. is targeted indicates a significant improvement in the situation of the families and their children from T0 to T2 [27]. By applying counterfactual analysis, the effectiveness of the

Table 3. ATE estimates

Variable	Kernel		Stratification	
	A	B	A	B
FAMILY-SERVICES RELATIONSHIP				
Parents' lower need from professional support	0,29 ***	0,84 ***	0,28 ***	0,34 ***
PROFESSIONAL EVALUATIONS				
CW scores				
Child's needs				
Health & Physical Development	0.19	0.20	0.16	0.14
Social Skills	0.58 **	0.69 ***	0.55 **	0.62 ***
Identity, Self Esteem & Social Presentation	0.57 **	0.61 **	0.53 *	0.53 *
Selfcare Skills	0.45 **	0.45 **	0.42 **	0.51 ***
Family & Peer Relationship	0.75 ***	0.82 ***	0.75 ***	0.73 ***
Learning	0.41 **	0.45 **	0.36 **	0.47 ***
Play & Free Time	0.75 ***	0.77 ***	0.75 ***	0.85 ***
Parental behaviour				
Basic Care	0.15	0.15	0.18	0.22
Emotional Warmth	0.30	0.33	0.34	0.38 *
Guidance & Boundaries	0.41	0.45	0.40	0.45 *
Play, Encouragement and Fun	0.65 **	0.66 **	0.65 **	0.68 **
Parents' Self-Realisation	0.74 **	0.76 **	0.70 **	0.83 ***
Environmental conditions				
Support from others	0.38	0.38	0.39	0.32
Participation in the community	0.44	0.42	0.44	0.45
Employment & Income	0.51 **	0.47 ***	0.46 **	0.57 ***
Housing	0.52 **	0.49 **	0.50 **	0.55 **
Relationship with Schools and Services	0.50 **	0.45 **	0.53 **	0.41 **
RISK factors				
Child	-0.63 **	-0.67 ***	-0.65 **	-0.69 ***
Family	-0.44 *	-0.47 *	-0.42 *	-0.51 **
Environment	-0.23	-0.22	-0.22	-0.32
PROTECTION factors				
Child	0.52 **	0.59 **	0.50 **	0.52 **
Family	0.48 *	0.56 *	0.46 *	0.53 *
Environment	0.49	0.52	0.44	0.46
Overall risk evaluation	-0.47 **	-0.58 ***	-0.41 ***	-0.55 ***

*** [**] (*) Difference statistically significant at the 99% [95%] (90%) level of confidence.

T(G-1) critical values considered, where G=12 is the number of territories.

Robust standard errors clustered by territorial area (bootstrapped for type A estimates).

Type A estimates: Simple difference of pre-post differences.

Type B Kernel estimates: Ordered Probit model for scores – regression of levels on D, T2*D and X=pre-treatment variables; Probit model for family situation – regression of the situation indicator on D and X=pre-treatment variables; values not comparable with the correspondent estimates in the other columns.

Type B Stratification estimates, 5 blocks: linear regression of the pre-post differences on D*block1, . . . , D*block5, propensity score *block1, . . . , propensity score *block5 (then the conditional ATEs are combined).

programme is confirmed on almost all outcomes. The impact seems to be particularly intense on the child's total risk of out-of-home placement and developmental needs. The effects on parents' responses to children's needs are instead dubious because of being statistically weaker and not confirmed by the application of different estimation methods. Anyway, despite the limited duration of the work with families, their overall situation at T2 improves, as parents' degree of "autonomy" from professionals' support increases. If on the one hand the risk factors decrease, on the other resources

and strengths significantly improve thanks to the programme.

This empirical evidence offers indications to professionals and all persons responsible for the programme implementation on how potentially modify practices and organizational aspects to achieve better outcomes. By relating the results of the counterfactual analysis with the process data on the individual micro-plans, questions arise about professionals' attention unbalanced to child's factors and the practical difficulties to discern and leverage on strengths and resources of parents and their relationships when, respectively, evaluat-

ing and working with families [28]. Furthermore, we expect that a longer duration for the intervention, whenever professionals deem it appropriate, could reinforce results, on parents' outcomes.

Doubts however arise in the interpretation of the impact evaluation results as well. Firstly, the data collected by professionals not directly involved in the programme (in the control group) is questionable respect to both quality and meaning. Secondly, the estimated impact, relative to an average effect over the entire treated group, does not consider inter-subjective variability in outcomes and in the intensity and type of heterogeneous individual interventions.

In the light of these considerations, the evaluation of P.I.P.P.I. with counterfactual approach is now moving on to the estimation of differential effects inside the programme itself. This strategy does not require to identify a control group of families not involved in the programme nor to gather additional data. Moreover, the rich amount of information already

available from the research instruments provided by the evaluation plan allows to better understand what kind of families are compared in the analysis, whether they are families with certain type of micro-plans or families that have taken advantage from specific activities.

In the research described in this paper the idea was to compare the programme with the standard practice of child protection, but the border between the two types of work can be very vague in some cases. It is indeed not possible to exclude a distortive contamination of the programme on the compared standard practices of social work with families and children. Consider, for example, the high percentage of control families whose professionals were previously trained in P.I.P.P.I. (73%). The specific actions characterizing the programme are also frequently observed in the control group, particularly regarding the relationship with schools, which results activated for more than 70% of non-treated families.

References

1. Pearl J. Causality. Models, reasoning and inference. – Cambridge, UK: Cambridge University Press, 2000.
2. Winship C., Morgan S. Counterfactuals and causal inference. – Cambridge, UK: Cambridge University Press, 2007.
3. Lacharité C., Ethier L., Nolin P. Vers une théorie éco-systémique de la négligence envers les enfants. – *Bulletin de psychologie* 59(4) (2006) 381-394.
4. Dubowitz H., Pitts S.C., Litrownik A.J., Cox C.E., Runyand D., Black M.M. Defining child neglect based on child protective services data. – *Child Abuse & Neglect* 29 (2005) 493-511.
5. Bronfenbrenner U. - The ecology of human development. Experiments by nature and design. – Cambridge: Harvard University Press, 1979.
6. Bronfenbrenner U. - Making Humans being Human. Bioecological perspectives on Human development. – London: Sage Publications, 2005.
7. Serbati S., Milani P. La tutela dei bambini. Teorie e strumenti di intervento con le famiglie vulnerabili. – Roma: Carocci, 2013.
8. Vaquero Tió E., Ius M., Paola M., Balsells Bailón M.A. Una revisión de la literatura sobre el uso de las TIC en el ámbito de la intervención sociofamiliar. – In: R. Roig-Vila (ed.), Tecnología, innovación e investigación en los procesos de enseñanza-aprendizaje. – Barcelona: Octaedro, 1920-1928, 2016.
9. Parker R., Ward H., Jackson S., Aldgate J., Wedge P. Looking after children: Assessing Outcomes in Child care. – London: HMSO, 1991.
10. Horwath J. - The Child's World: the Comprehensive Guide to Assessing Children in Need. – London: Jessica Kingsley Publisher, 2010.
11. Serbati S., Ius M., Milani P. P.I.P.P.I. Programmeme of Intervention for Prevention of Institutionalization. Capturing the evidence of an innovative programmeme of family support. – *Revista de cercetare si interventiesociala* 52 (2016) 26-50.
12. Braconnier V., Humbeeck B. L'évaluation de l'état de danger dans la famille. – In: Rapport intermédiaire de recherche. – Mons: Université de Mons-Hainaut/CERIS, 2006.
13. Milani P., Colombini S., Di Masi D., Ius M., Santello F., Serbati S., Sità C., Tuggia M., Zanon O. P.I.P.P.I. Programmema di Intervento Per la Prevenzione dell'Istituzionalizzazione. Rapporto di valutazione. Sintesi 2015-16. – *Quaderni della ricerca sociale* 39 (2017).
14. Holland P.W. - Statistics and causal inference. – *Journal of American Statistical Association* 81(396) (1986) 945-960.
15. Heckman J. Instrumental variables: A study of implicit behavioral assumptions used in making programme evaluations. – *Journal of human resources* 32(3) (1997) 441-462.
16. Rosenbaum P.R., Rubin D.B. Constructing a control group using multivariate matched sampling methods that incorporate the propensity score. – *The American Statistician* 39(1) (1985) 33-38.
17. Heckman J., Ichimura H., Todd P.E. - Matching as an econometric evaluation estimator. – *Review of economic studies* 64(4) (1997) 605-654.
18. Heckman J., Robb R. Alternative Methods for Evaluating the Impact of Interventions. – In: Heckman J. and Singer B. (eds.), Longitudinal Analysis of Labor Market Data. – Cambridge, U.K.: Cambridge University Press, 1985.
19. Abadie A. - Semiparametric difference-in-differences estimators. – *Review of Economic Studies* 72 (1) (2005) 1-19.
20. Heckman J., Robb R. - Alternative Methods for Evaluating the Impact of Interventions. – In: J. Heckman and Singer B. (eds.), Longitudinal Analysis of Labor Market Data. – Cambridge, U.K.: Cambridge University Press, 1984.

21. Rosenbaum P.R., Rubin D.B. The central role of the propensity score in observational studies for causal effects. – *Biometrika* 70(1) (1983) 41-55.
22. Imbens G.W. - The role of the propensity score in estimating dose-response functions. – *Biometrika* 87(3) (2000) 706-710.
23. Becker S., Ichino A. Estimation of average treatment effects based on propensity scores. – *The Stata Journal* 2(4) (2002) 358-377.
24. Imbens G.W., Wooldridge J. M. - Recent developments in the econometrics of programme evaluation. – *Journal of economic literature* 47(1) (2009) 5-86.
25. Leuven E. and Sianesi B. Psmatch2: Stata module to perform full Mahalanobis and propensity score matching, common support graphing, and covariate imbalance testing. – [<http://ideas.repec.org/c/boc/bocode/s432001.html>], 2003.
26. Serbati S., Santello F., Colombini S., Milani P. Challenges for the evaluation of the P.I.P.P.I. Programmeme of Intervention for Prevention of Institutionalisation: between participative and experimental pathways. – *Interdisciplinary Journal of Family Studies* 21(2) (2016) 1-24.
27. Santello F., Colombini S., Ius M., Milani P. P.I.P.P.I.: What has changed? How and why? The empirical evidence. – *Rivista Italiana di Educazione Familiare* 2 (2017) 111-136.
28. Serbati S., Ius M., Milani P. P.I.P.P.I. Programmeme of Intervention for Prevention of Institutionalization. Capturing the evidence of an innovative programmeme of family support. – *Revista de cercetare si interventie sociala* 52 (2016) 26-50.

Understanding the Importance of Social Support Networks: Integration Process of Migrants

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Abstract. This article examines the state of knowledge concerning the different types of social networks that exist for the migrants to assist them to be integrated into a new country at local level. It displays the kind of support they offer and reflects on the significant role that they play towards their effort to build a "new" life. The main concepts and debates are presented aiming to understand their role in social and economic integration, the influence of inter-ethnic contact and how policy and legislative contexts should include them when designing multi-cultural coexistence as well as integration policies that prevent tensions.

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Short title: Integration process of migrants.

Introduction

One of the greatest challenges of the recent time is managing global migration flows across the borders. Particularly into the countries of European Union (EU), over the last decade, sharply increasing number of migrants has become a challenge to European solidarity: in terms of both a core social principle as well as 'shared responsibility' between the governments and the civil society to help people in need by adopting a humanitarian perspective. The attention is focused on the need to actively integrate migrants into the host societies as the most effective way to become both: actors and partners into a society development. The Council of the European Union indicates [1] that the integration of third-country nationals in the Member States is a key element in promoting economic and social cohesion.

A huge number of actors worldwide are looking for ways how to make migration efficiently developing, and at the European Union level this has been translated into an increased embedding of migration in EU development policies, with increasing awareness of the importance of migration as a key development factor. It is considered that the only way to meet the challenges which migration brings is by working together, by initiating the core services and infrastructure migrants' formal and informal social networks.

Social networks are widely recognised to be influential on people's lives as they fulfil different functions [2]. They provide either direct or indirect benefits and have the potential to supply tangible resources and contacts [2-4]. With regard

to the migration, their values and importance are identified in multilevel facets: they provide information on the migration process itself, support migrants in the first steps of their integration process and have an influence on the perspective of migrants' residence strategies evolve over time [5, 2, 6-8]). Assessing the relative importance of these roles is crucial to understand the trend of migration and to the design of relevant immigration policies.

The paper discusses the types of networks that are created for the migrants to help them integrate into a new country, the kind of support they offer and reflects on the significant role that they play towards their effort to build a 'new' life in an EU member state. The entire intention is to better understand the expectations, anticipations and strategic reactions of migrants in relation to immigration policy that has been founded in EU territory.

1. Types of Networks

Each person has a certain number of family members and close friends, who are regularly in touch with one another and it is this type of connection which forms a network consisting of ties [4, 9]; if the members have developed strong bonds, their network is identified that has strong ties. Everyone is also in touch with a certain number of people (acquaintances, colleagues, relatives' acquaintances) who usually do not know each other and who are connected to one another by a set of weak ties [9].

Both types of networks are valuable. The personal net-

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works, quite often, the existence of family bonds can (and usually does) help the newcomers to adjust to the new environment. However, attention must be given to strengthen the weak ties of social networks. Those networks can bridge the gap between separate personal networks as they can provide access to resources and information beyond those directly available to them [4].

Woolcock and Narayan [10] argue that people with extensive and diverse networks are in a better position than those who lack them. They acknowledge that a dynamic process exists which combines the different types of networks among contexts and individuals [10]. Some migrants can activate certain networks to obtain the resources they need, depending on the social networks they have and their individual skills. It is these cases, which make many officials to believe that all migrants have known each other in the country they live. In contrast, it has been observed that when a migrant lacks extensive networks, they strive much harder to 'get ahead' and improve their status over time [5, 8, 11-12].

The technological progress which enabled cheap and efficient global connections along to the increased transnational flows of migrants from same nationalities or groups, increasingly enable them to maintain and develop social relations across distant locations [13]. It is those border-crossing that boost and promote social relations and favour the establishment of transnational networks which help the newcomers during the first period in the host country. The existence of network influence also the migrant's ability to move to a destination, find a job and housing, open a business, participate in the development of their home country, and access health care can all be directly impacted by or even dependent upon the migrant's social network. The migration flows also demonstrate that about half of all movements across international borders take place where large groups of people from the same country of origin or neighbouring countries are living. People do not simply look around the world and arbitrarily decide where they might like to pick up and relocate. Most potential immigrants seek to minimize their risks when they move and consider places where they know other individuals or organizations that can help them to make the trip and settle most easily. Social networks provide the kinds of connections needed to make migration possible.

It is important to understand the value of social networks as they connect communities together. Zitek and Hebl [14] argue that social networks play an important role in attitude formation and/or change. They create social norms within which individual cognitive processes take place. Shifting then the discussion to the migration it is argued that they shape attitude formation because they provide the context within which individuals process messages about asylum and immigration they may either reinforce or undermine the attitude formation about the discussed item [15].

The different sources of support they may receive from their networks are displayed in the following section.

2. Sources of Support

Sources of support are usually less clear. Portes [12] argues that such interactions have four potential sources. People agree to act in a certain way because they have internalised certain social norms and values; it is the case in which support is underpinned by strong collective values [5]. Variation in the nature of the actors within a network and of the contents of the bonds linking them influences the form of network activities and solidarity displayed. Social relationships are progressively transformed into social obligations, and well-settled migrants feel indebted to assist newcomers, because it is a norm in their society. In this case, support is based upon personal values. This perhaps explains why certain people or institutions assist certain groups that have been socially marginalized.

When expressions of support emerge from the feeling of sharing common situational circumstances, support is the result of a bounded solidarity [16]. People who share a common fate identify with one other and support each other's initiatives [5].

Solidarity is not necessarily bounded by the limits of the ethnic group or by a nationality. When the motivation for making resources available to migrants is the expectation of payback, support is driven by instrumental motives. When support groups give access to the resources expecting them to be fully repaid in future, exchanges are based on the norm of reciprocity.

3. Service provision via formal and informal channels

The groups that support migrants with basic services are both formal and informal in nature. There may exist formal support groups of origin, created in response to migrants' needs, and acting in accordance with a specific mandate. There are also cases in which they setup membership organizations that provide the assistance to their members, while others provide support regardless of membership. Other formal institutions are not "ethnic-based" and instead base their support on a certain view on human rights. NGOs act according to the principle that everyone, as a member of society, has the right to a minimum standard of living, which includes food, clothing, housing, medical care, and basic social services.

Other support mechanisms are not part of any institutionalised process; they simply emerge from an encounter with someone in difficulty. They are informal in nature. It is usually their own personal history that led them to help other migrants from their country of origins [5]. Expressions of support emerge from the feeling of having shared; if not an entire past, at least certain parts of it. This generates a certain form of bounded solidarity [12].

However, solidarity is bounded by the limits of each migrant community at large, and crosses ethnic groupings, lan-

guage and countries of origin [5].

The good case scenario is observed when one ethnic group is willing to help the other ethnic groups. Although they may have, or still face similar difficulties, the level of support may not exist. Furthermore, even in cases where support is provided, a limited duration of support is, in several cases, observed. Across countries, newcomers are helped by peers for a few days, and then the responsibility is transferred to someone else. That reality also explains why those newcomers have little choice but to rely on the goodwill of strangers (i.e. from locals and activists) at the beginning of their stay, which can have positive effects on their lives, or, in contrast, they can be wrongly informed and take the wrong decisions for the regularisation of their lives in Europe.

Finally, there is the case in which the State plays a significant role. It is often observed the non-governmental channels of support to play an increasing role in 'providing the basics' to this fringe of the population. It is acknowledged that their role in the field of pro-migrant programmes cannot be compared to the role of volunteers and informal networks. Their intervention is characterised in reference to the original identity of a non-profit organisation and to the cultural values related of the community norms from which they originate. Such situations challenge the professional values and ethics of both individuals and professionals. A constant threat for the sustainability of providing services and response to migrant needs prohibits the continuation in planning an active integration. Given that support groups have both limited resources and a limited mandate, they cannot afford to offer long-term assistance to everyone in difficulty. Their main contribution is to offer short-term relief and to provide contacts, assuming migrants will eventually meet someone who will be able to help them for a longer period. On the other hand, there seem to be few alternatives for migrants who fail to secure good contacts [17]. If they fail to attain a sustainable livelihood, they will continue to rely on charity, move to the other places or even to return to their countries of origin.

Conclusions

It has been shown in this article the different types of social networks that exist for the migrants to become integrated into a new country. Positive and negative aspects of the existing channels of support reveal the incompatibilities in the integration process. It is suggested that unless intensive and specialised planning and comprehensive social inclusive po-

licy is introduced and organisations are prepared to support such initiatives, the integration will remain limited.

Communication and activities combine both sides – foreigners and most indigents contribute to a harmonious co-existence and the prevention of tensions. Social networking and the development of dialogue between cultures and religions is necessary for promoting tolerance, as well as for eliminating racism and xenophobia and other negative phenomena. It is also important to understand the difficulties migrants face in the host countries at multiple levels. The absence of a coherent supportive network to assist their initial settlement to the new country increases the possibilities of negative attitudes and misunderstandings of responsibilities. Non-governmental channels of support play a significant role in providing the assistance to migrants in difficulty. Support is based on bounded solidarity, humanitarian principles, or on strong personal values. Individuals and NGOs act in accordance with the certain view on human rights and rely on the principle that everyone has the right to a minimum standard of living, regardless of their being a legal resident or not.

Further research is called for the exploration of hypothesis that there is a shift from state to non-state actors. If there is a shift, non-governmental actors must be given the necessary means to relay the State efficiently. It has also to be noted that shifting the burden of unwanted migration to non-governmental groups is not a new trend. It has been observed that in European policy making for nearly a decade now, a systematic effort to actively involve NGOs into the migration, by limiting at the same time any public expenditure. However, it cannot be called a shift, if they are not given sufficient resources towards this direction. It merely comes down to the failure to protect a whole segment of the population that is still within the state boundaries, and it flings open the door to social exclusion.

It is argued that unless both the organisations and professionals re-examine their value basis, the integration efficiency will remain limited. Since integration has been a two-way process, which is supposed to be based on the will to integrate and on mutual benefits for both parties, it is necessary to encourage migrants to actively participate in common societal activities. In times of political uncertainty, a need to build alliances is more than a theoretical plausible scenario. The above-mentioned circumstances call for learning a lesson about social solidarity in responding to global crises in more sensitive and culturally relevant manner.

References

1. Council of Europe. Council Decision on establishing the European Fund for the Integration of third-country nationals for the period 2007 to 2013 as part of the General programme Solidarity and Management of Migration Flows, 2007/435/EC: 25 June 2007. – <<https://publications.europa.eu/en/publication-detail/-/publication/569026ce-8f9b-4610-9665-3386a0cdee7f/language-en>>.
2. Dolfin S. and Genicot G. What Do Networks Do? The Role of Networks on Migration and "Coyote" Use. – *Review of Development Economics* 14 (2006) 343–359.

3. Bourdieu, P. 'The forms of capital', in Richardson, J.G. (Ed.), *Handbook of Theory and Research for the Sociology and Education*, New York: Greenwood Press, (1986) pp.:241–258.
4. Granovetter M. The strength of weak ties: a network theory revisited. – *Sociological Theory* 1 (1983) 201–233.
5. Chelpi-den Hamer M. and Mazzucato V. The Role of Support Networks in the Initial Stages of Integration: The Case of West African Newcomers in the Netherlands. – *International Migration* 48 (2) (2010) 31-57
6. Alitolppa-Niitamo A. Somali youth in the context of schooling in metropolitan Helsinki: framework for assessing variability in educational performance. – *Journal of Ethnic and Migration Studies* 30(1) (2004) 81–106.
7. Engbersen G. and J. van der Leun . The social construction of illegality and criminality. – *European Journal on Criminal Policy and Research* 9 (2001) 51–70.
8. Guarnizo L.E., Portes A. and Haller W. Assimilation and transnationalism: determinants of transnational political action among contemporary migrants. – *American Journal of Sociology* 108(6) (2003) 1211–1248.
9. Henning C. and Lieberg M. Strong ties or weak ties? Neighbourhood networks in a new perspective. – *Scandinavian Housing and Planning Research* 13(1) (1996) 3-26
10. Woolcock M., and Narayan D. Social capital: implications for development theory, research and policy. – *World Bank Research Observer* 15(2) (1999) 225–249.
11. Palloni A., Massey D.S., Ceballos M., Espinosa K. and Spittel M. Social Capital and International Migration: A Test Using Information on Family Networks. – *American Journal of Sociology* 106(5) (2001) 1262-1298. – DOI: 10.1086/320817, available online <<https://www.jstor.org/stable/10.1086/320817>>.
12. Portes A. Social capital: its origins and applications in modern sociology. – *Annual Review of Sociology* 24 (1998) 1–24.
13. Castle, S. and M.J. Miller. (). *The Age of Migration: International Population Movements in the Modern World*. Fourth edition. – New York and London, the Guilford Press, 2009.
14. Zitek E.M. and Hebl M.R. The role of social norm clarity in the influenced expression of prejudice over time. – *Journal of Experimental Psychology* 43(6) (2007) 867-87.
15. Levitan L. C., Clarke P. S. The impact of the social context on resistance to persuasion: Effortful versus effortless responses to counter-attitudinal information. – *Journal of Experimental Social Psychology* 44 (2008) 640–649.
16. Portes A. and Sensenbrenner J. Embeddedness and Immigration: Notes on the Social Determinants of Economic Action. – *The American Journal of Sociology* 98(6) (1993) 1320-1350
17. Christou A. Agency, Networks and Policy: The Case of Poles in Greece. – *Journal of Immigrant & Refugee Studies* 6(3) (2008) 312-325

Migration in Lithuania: Children Left Behind

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Abstract. Regarding the context of globalization and various life changes in families, including losses that people experience (i.e. migration, divorce, death of a loving ones etc.), the socialization of children who have experienced loss in the family is explored in this research. Some behavioural and emotional differences are revealed as well, showing that grieving children (especially after facing parent's migration, death, or divorce) more often experience negative emotions (sadness and anger), and less the positive ones. The reasons of positive emotions for such children are social ones (teachers', family members' support etc.). However, negative emotions arise often from personal and sometimes – neutral – reasons. Also, such children, who experience more positive emotions, are quite often linked to be self-confident and respectful, and those, who experience negative emotions, are tended to be withdrawn, less self-confident, and sensitive. Besides, the data has shown that grieving children could be characterized as less respectful to the others, peaceful, sensible and self-controlled than others. Thus, grieving children's social-moral behaviour is less stable than those who have not experienced the loss.

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Short title: Migration in Lithuania.

Introduction

Most of the socialization's research is directed towards the individual, as social being's development and his/her participation in society. Thus, the socialization becomes an important factor of an individual's participation in the society, and the expression, according to which, the relations with surrounded environment and its events, and phenomena are revealed. On the other hand, socialization is quite a difficult psychosocial process: in a fast-developing society, it becomes harder for children to socialize adequately.

This paper was prepared using results working under the project *European Citizens for Solidarity - EUROSOL* [1] in 2017.

Various social changes (sudden growth of technologies and science, mobility, urbanization, individualization etc.) have an influence on the participants and determinants of socialization [2]. According to J. Gibson and P. Blumberg [3], adults are influenced by those changes directly through their experienced anxiety and frustrations. However, children are mostly influenced by those ways that were important to adults also. Consequently, the socialization process is strongly linked with a child's need to identify himself and internalize those values that are appropriate to society.

As C. Thornton [4] conceptualizes, childhood is the time of learning from various experiences. This is a time, especially beginning with the school year, when child develops cognitively, physically, and socially [2]. Thus, every single moment in childhood is useful, even if it would be the loss of a loved one. The period of 7–11 years in the childhood is mainly stressed as the school life comes into child's social field. Particularly, the relation between loss in the family and child socialization is stressed in many psychological works [5]. Experienced loss is a very difficult matter for a child, who tries to internalize his/her family traditions, values and to form his/her identity - *self I*. After the loss, a child loses the object of identification *with whom*. As researches show, not satisfied need of identification can laden the value internalization as well [6]. To put it in other words, boys, who have lost their fathers and girls who have lost their mothers, partly lose self-identification as a man, and as a woman.

The recent researches have shown that children from incomplete families are more linked to the aggressiveness than others from nuclear families [7-8]. On the other hand, children, age 6–7 and even younger, after the loss of one of the parents, experience guilt for the things that have happened [9]. All this may put children at risk to lose *elf-core* and on-

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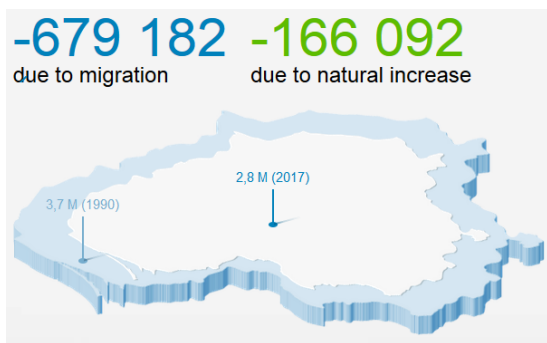


Fig. 1. Population in Lithuania. Adapted according to Ref. [15].

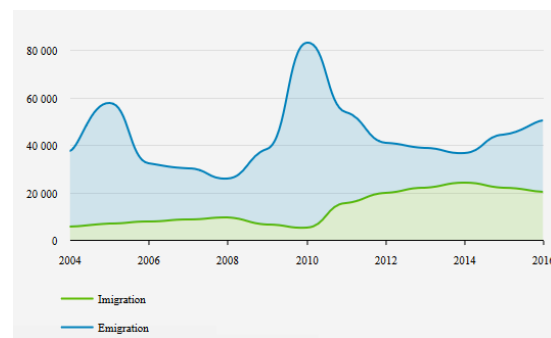


Fig. 2. Migration flows in Lithuania. Adapted according to Ref. [15].

tological safety. Thus, the loss in the family with its negative emotional charge also makes it more difficult for a child’s need of identification, and consequently – value internalization. In fact, the loss makes it rather complicated for such children’s socialization processes. Besides, the experienced loss (parents’ divorce, death, unemployment or migration, grandparents’ death etc.) has more negative influence on children’s emotional, cognitive and behavioral spheres. Speaking about such children’s emotions, much research [8, 10-12] reveals that these children emotionally are much more vulnerable than those, who did not experience the loss.

The research object is the behavioural and emotional differences of children, who have experienced loss in the family and those who haven’t in a case of parent migration.

The aim is to reveal the socialization peculiarities based on behaviour and emotions level of such children.

1. Migration: digitalized view

Population in Lithuania has been declining: from 3 million in the 2011 census to the estimated 2.85 million at the beginning of 2017 [13,14]. Fig. 1 shows decreasing numbers of Lithuanian population within past two decades. It becomes obvious that people migrate because of various reasons but the main remains the economic one.

Migration flows reveal the tendencies of how many immigrants and those who leave the country exist in the period of 2004 and 2016. As we may observe (see Fig. 2), there are

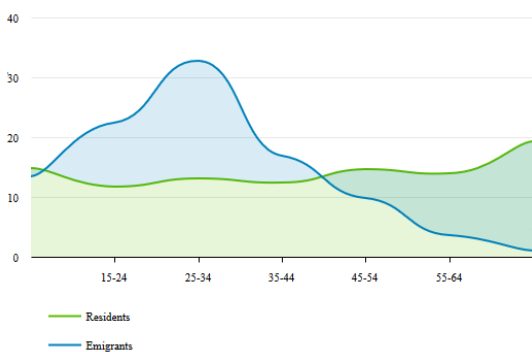


Fig. 3. The balance between immigrants and migrants in Lithuania. Adapted according to Ref. [15].

rather an disbalanced numbers of those coming into the country and those who leave. Such statistical data could serve as great both economic and psychosocial indicators while analyzing the factors of such flows.

The number of foreign nationals residing in Lithuania increased by 0.8% over 2016 to 44 600 at the beginning of 2017 [14]. In Lithuania (period of 2015-2016-2017), the number of emigrants outnumber the number of immigrants [15].

Fig. 3 represents the balance between immigrants and migrants in Lithuania. The percentage shows the main tendencies of migration flows in the country. Referring to the Fig. 2, we may interpret both economic and psychosocial reasons and impact of such migration that have happened and still happens in Lithuania.

Fig. 4 represents the amount of migration. The data presents on how many Lithuanians would choose some countries – as target places – for their future residence. As we may see, most popular target countries remain United Kingdom, Germany, Norway, and Ireland.

2. Socialization process during the childhood

The socialization is rather a difficult psychosocial phenomenon along with values internalization and self-identification mechanisms, and this process is influenced by other social factors that begin in early childhood and last through the lifespan [2,16].

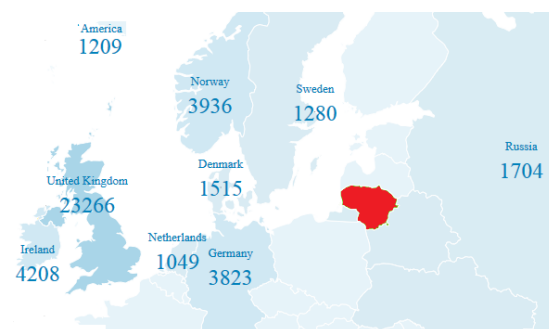


Fig. 4. Amounts of migration. Adapted according to Ref. [15].

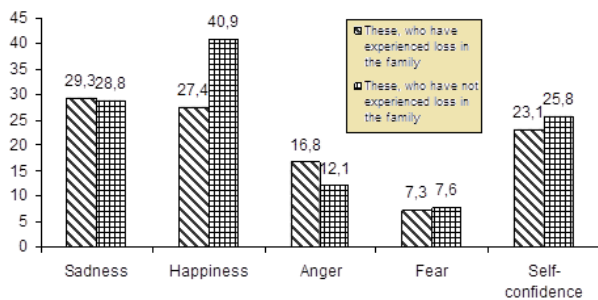


Fig. 5. Grieving and non-grieving children emotions.
Adapted according to Ref. [5].

The socialization in childhood starts as a child's full enrolment into the society, taking all the social roles. Besides, the constant influence of the environment in many ways helps child to understand and internalize socially accepted values and to identify his/her place in the context of various social interactions. Speaking on the factors, that influence children's socialization, we may state that essentially both internal (psychological, cognitive, physiological) and external (social, educational) factors undoubtedly influence children's socialization [16].

However, the external factors (family, school, peers etc.) are more obvious and much more important currently for children as they get increasingly involved in various groups' activities [2, 5]. Although the internal socialization factors are important as well: the rapid development of logical thinking, comprehension, imagination, attention, and memory [17]. All these components play rather important role for child's socialization tendencies and success. Aside there are not only purposeful, but hardly predictable both internal and external socialization factors that children experience.

Family's role is very important as well as family is one of the essential sources of socialization that leads to child's full enrolment into the society and appropriate values' internalization [16, 18]. The success of values' internalization mostly depends on what kinds of relations are established within the family.

On the other hand, these relations quite often are troubled, because of many crises that the family nowadays are exposed to. One of such crises is the rapid spreading of various losses that child experiences. Thus, the loss in the family can be harmful for children's socialization [11, 19]. In the context of globalization there are many kinds of experienced losses within the family, but mostly explored are: changing the living place, losing the parent's rights, family conflicts, parent's divorce and one of the parent's death.

3. On grieving children's behaviour and emotional differences

During the research it was set that grieving children less than the others internalize openness, believing in self and others, sensitiveness, respect ($\chi^2=42.39$, $p<0.002$),

self-control ($\chi^2=87.52$, $p<0.0001$), and activity ($\chi^2=31.67$, $p<0.005$). However, they came up as more solidary. Boys in this case were affected more than girls by the experienced loss in the family [5]. The results have confirmed that grieving boys less than the others internalize the openness ($\chi^2=84.14$, $p<0.0001$), believing in self and others, sensitiveness ($\chi^2=69.18$, $p<0.0001$), respect, self-control ($\chi^2=33.16$, $p<0.03$), solidarity ($\chi^2=29.23$, $p<0.02$), and activity. These values are more obvious within non-grieving children's behaviour [5].

Meanwhile grieving girls' behaviour in many cases differs from that of boys and other classmates. The data shows that grieving girls quite more than the others – non-grieving classmates – practically internalize solidarity, respect, activity, and openness. Although, believe in self and others is less internalized value among those girls' behaviour.

The research data also reveals that grieving primary school children more often than other classmates experience negative emotions (sadness and anger), and less – positive ones (happiness and self-confidence), particularly those, who have experienced parents' divorce or their death. Boys, opposite than girls, are less self-confident (see Fig. 5).

During the research it was obvious that positive emotions for the grieving and non-grieving children arise more often from socially orientated causes than personally ones, and negative due to personal reasons, less neutral and partly to social causes. The difference between those children is that socially oriented causes stimulate self-confidence for the grieving children, and less happiness [5].

Also, data reveals that grieving children hardly recognize other people's emotional expressions, i.e. happiness, self-confidence, and anger. However, they recognize sadness easier than others. Meanwhile, for the non-grieving children it was difficult to recognize sadness but quite easier happiness.

Conclusions

There are many life experiences that can cause feelings of grief in a child: from the death of a relative or a divorce in the family to more everyday experiences such as moving to a new neighbourhood, other country or losing a prized possession. Socialization is quite multidimensional psychosocial phenomenon that is more often described as individual's involvement into the social life and internalizing socio-moral values through the acceptance of social roles. However, the experienced loss in the early years may influence negatively the socialization process, complicates the internalized values based on child's behaviour and emotions.

The research has shown some negative behavioural peculiarities on the level of internalized values and emotions of those primary school children who have experienced the loss in the family. In many cases the identification processes for the grieving children are laden, and they are often linked to

internalize those values that are not acceptable to the society they are living in.

Grieving children more often experience negative emotions (sadness and anger), and less – the positive ones. The reasons of positive emotions for such children are social ones (teacher support etc.). However, negative emotions arise often from personal and sometimes neutral reasons. Besides, grieving children have more difficulty recognizing happiness and self-confidence, experienced by others, but they recognize sadness easier than non-grieving classmates. Chil-

dren, who experience more positive emotions, are quite often linked to be self-confident and respectful, and those, who experience negative emotions, have tendency to be socially withdrawn, less self-confident, and sensitive.

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References

1. EUROSOL, European Citizens for Solidarity project. The project involves European partners from Italy, Bulgaria, United Kingdom, Germany, Poland, Lithuania, Cyprus and the General Directorate of Social Services and Social Integration of the Regional Government of Madrid (Spain). EUROSOL main goal is to establish the European network among entities very active in the field of solidarity and volunteering programs. Also EUROSOL is directly related to the European Solidarity Corps Initiative adopted by the European Commission last December 2016. This project will run until the 30th September 2018 and during this period more than 750 young people will benefit directly and about 16,000 will benefit indirectly.
2. Berk L. E. Child Development. 7th Ed. – Boston: Pearson Education, Inc., 2006.
3. Gibson J. Blumberg P. Growing Up: Readings on the Study of Children. Reading, Mass. – Addison-Wesley, 1978.
4. Thornton C. M. Using Children's Literature to Help the Grieving Child,(2001) <<http://kahuna.merrimack.edu/cthorton/htm>>, accessed 2017 12 10.
5. Butvilas T. Šeimoje netektį patyrusių vaikų socializacija. Monography (in lith.) – Vilnius: MRU leidykla, 2008.
6. Perry B. D. Children & Loss. Vol. 6. – New York: Instructor, 2001.
7. Goldman L. Helping the Grieving Child in School. – *Phi Delta Kappa Fastbacks* 460 (2000).
8. Worden J. W. Children and Grief: when a parent dies. – New York: The Guilford Press, 2001.
9. Mishara A. L. Narrative and psychotherapy – the phenomenology of healing. – *American journal of psychotherapy* 49(2) (1995) – DOI: 10.1176/appi.psychotherapy.1995.49.2.180.
10. Kübler-Ross E. On children and death. – USA: First Touchstone Edition, 1997.
11. Webb N. B. Helping Bereaved Children. 2nd Ed. – New York: The Guilford Press, 2002.
12. Tomer A. et al. Existential and Spiritual Issues in Death Attitudes. – New York, London: Lawrence Erlbaum Associates. Taylor&Francis Group, 2008.
13. EMN: <https://ec.europa.eu/home-affairs/sites/homeaffairs/files/17a_lithuania_national_policy_report2016_part2_lt.pdf>, accessed 18-01-2018.
14. OECD: <<http://www.oecd.org/migration/international-migration-outlook-1999124x.htm>>, accessed 12-12-2017.
15. EUROSTAT: <<http://ec.europa.eu/eurostat/web/population-demography-migration-projections/migration-and-citizenship-data>>, accessed 23-11-2017.
16. Berns R. Vaiko socializacija: šeima, mokykla, visuomenė. – Kaunas: Poligrafija ir informatika, 2009.
17. Piaget J. The Moral Judgment of The Child. – New York: Free Press, 1965.
18. Chapman G., Campbell R. Penkios vaikų meilės kalbos. (in lith.) – Kaunas: Sidabrinis trimitas, 2001.
19. Benedict H. Bereavement in Children. Mining Report. – Association for Play Therapy (USA), 2008.

