Zipf and Related Scaling Laws. 1. Literature Overview of Applications in Economics

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Abstract. Zipf law is well-known by modeling the economic human activity. Power law distributions of such type (so called zipfian) through parametrization are related to the Pareto distribution. This review is devoted to analysis of application of scaling law in economics. The six themes are observed: company size and bankruptcy, wealth distribution, systems of finite recourses, investment strategy, trading and stock market models, city creation mechanism and driving forces for city expansion.

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Introduction

Power-law dependencies are very large distributed in the world. Many sets of data studied in economical and natural sciences can be approximated by the dependence where probability is inversially proportional to the item rank. Many factors in economics such as executive pay, income, trading volume, international trade, wealth, stock market returns, the size of cities and firms etc are surprisingly distributed according to power law with exponent equal to 1.

As an example, power law is well known in linguistics. Items of large regular texts written in any human language are distributed not randomly but follow a power law.

George Zipf [1] found that frequency $f(r)$ of item (word) occurrence in finite corpus is inversely and linearly related to item rank $r(w)$ – so called **Zipf** law, see Eq.(1).

$$f(r) = \frac{\alpha}{r^{\gamma}} \tag{1}$$

Benoit Mandelbrot [2] proposed the generalized expression of Zipf distribution as a discrete probability distribution – so called **Zipf-Mandelbrot** distribution.

$$f(r) = \frac{\alpha}{(1+\beta r)^{\gamma}} \tag{2}$$

$$f(r) = \frac{\alpha}{\beta + r^{\gamma}} \tag{3}$$

Both distributions contain the adjustable parameters α, β, γ which are item content-dependent. In particular case for ranked frequency $f(r)$ of item occurrence in finite English corpus, Zipf distribution parameters are following: $\alpha \approx 0.1$, $\gamma \approx 1$ (Eq.(1)). Fig.1 represents idealized single-linear Zipf dependence in log-log scale.

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In theory of statistics, the **Yule–Simon** distribution is a discrete probability distribution.

$$f(r) = \frac{\alpha \cdot \beta^{r}}{r^{\gamma}} \tag{4}$$

Eq.(4) represents the limiting distribution of a particular stochastic processes which was studied by Udny Yule as a distribution of biological objects. Herbert A. Simon [3] rationalized mentioned compound distribution where the parameter of a geometric distribution was treated as a exponential function.

Exponential function – see Eq.(5) – is useful for comparative analysis when function growth (decay) rate is proportional to the function argument. In many cases exponential model could be treated as the fundamental due to relations to the Gauss normal distribution.

$$f(r) = \frac{\alpha}{\exp(\gamma r)} \tag{5}$$

For multi-level complex system, second generalization of Zipf law was realized as well-known **Menzerath-Altmann** equation [4] – see Eq.(6).

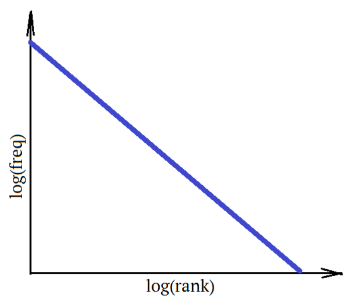

Fig. 1. Frequency dependence on rank. Idealized single-linear Zipf distribution in log-log scale - see Eq.(1), γ=const.

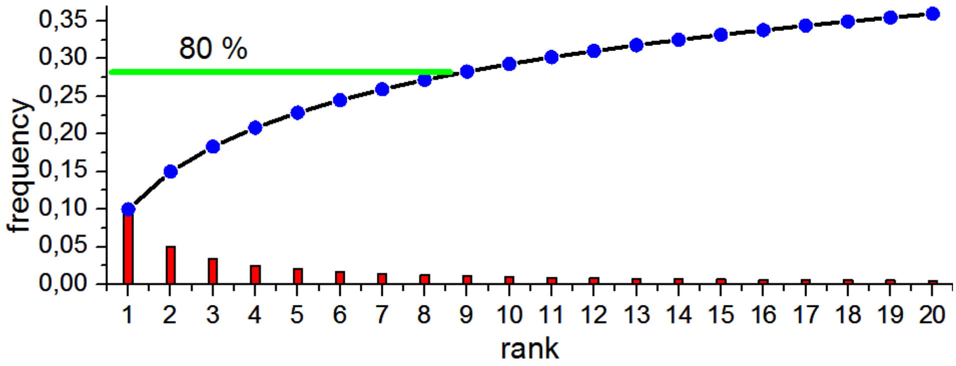

Fig. 2. Frequency dependence on rank. Idealized single-linear Zipf distribution, Eq. (1), α=1, γ=1. column bar, red. Pareto chart, blue. 20% of products gives 80% of profits (Pareto rule).

$$f(r) = \frac{\alpha \cdot r^{\beta}}{\exp(\gamma r)} \quad (6)$$

Gibrat law claims that size of firm and its growth rate are independent [5]. Vilfredo Pareto observed in 1906 that 80% of the land in Italy was owned by 20% of the population. The **Pareto** principle (also known as the principle of factor sparsity, so-called factor scattering or 80/20 rule) states that, for many events, roughly 80% of the effects come from 20% of the causes.

$$f(r) = \left[\frac{r}{r_{min}}\right]^{-\gamma} \quad (7)$$

Fig. 2 represents Zipf distribution (column bars represent the ranked frequencies - individual values in descending order) and Pareto chart (line graph represents cumulative total index). The purpose of Pareto chart is to highlight the most important amount of ranked factors. In many cases, 80% of content could be titled as significant. Pareto chart belongs to the famous tools of quality control.

Gutenberg–Richter law [6] expresses the relationship between the magnitude M and total number of earthquakes N in any given region and time period.

$$\log_{10} N = \alpha - \beta \cdot M \quad (8)$$

Eq.(8) was derived originally in seismology from empirical data. Modern attempts in explanation are grounded on self-organized criticality.

Hill estimator [7] is a popular method for estimating the thickness of heavy tails. Approximation of the distributional tail must be provided with a power function. In practice it is often true equation Eq.(9) for x>0:

$$P(X > x) \approx Cx^{-\gamma} \quad (9)$$

Then the idea is to estimate the parameters $C > 0$ and $\gamma > 0$ by a conditional maximum likelihood estimate based on the r+1 (0<r<N) largest order statistics, which represent only the portion of the tail for which the power law approximation holds.

Usage of Hill estimator in some cases is sophisticated but sometimes it is necessary due to so called robustness of dependencies. Since it only depends on the shape of the probability tails, it can be applied in situations where the form of the distribution is unknown. This is typically the case in applications to finance, where heavy tails are common.

Zipf law in economics is well-known by modeling the ranked firm size distribution, income-wealth distribution, city size distribution etc. Power law distribution of such type (so called zipfian) through parametrization are related to the Pareto distribution. This review is devoted to analysis of application of scaling law in economics. Six themes of big importance are observed here:

1) company size and bankruptcy;
2) wealth distribution;
3) resourses and investment strategy;
4) trading / stock market models;
5) city creation mechanism;
6) driving forces for city expanding.

1. Company size and bankruptcy

Economic prosperity is determined by the activity of the firms. Firms, stock companies, corporates etc are established to achieve certain economic goals for a certain period of time. Creation, growth, prosperity, stagnation, and bankruptcy - these states of company describes the natural way of development, which influences the macro-economic indicators. Although firm growth and bankruptcy are stochastic processes, they could be forecasted by analysing dynamical tendencies in certain economic area.

Ausloos et al. [8] state that many problems in economy and finance is possible to solve using methods of statistical physicists. Presence of financial cycles and existence of power-law correlations in economic systems allow to use digitalized methods such as fluctuation analysis, multi-component analysis etc. The well-known financial analyst technique, moving average, is shown to raise questions about fractional brownian motion properties. Also Zipf method is useful for sorting out short range correlations.

Wright [9] represent the self-organized dynamic model of the social relations between workers and capitalists. Several empirical distributions were used: power-law firm size distribution, the Laplace firm and GDP growth distribution, the lognormal firm demises distribution, the exponential recession duration distribution, the lognormal–Pareto income distribution, and the gamma-like firm rate-of-profit distribution. In the framework of model, these distributions are interconnected in order to generate the business cycle phenomena. The generation of an approximately lognormal–Pareto income distribution and an exponential–Pareto wealth distribution demonstrates that the power-law regime of the income distribution can be explained by an additive process on a power-law network that models the social relation between employers and employees organised in firms, rather than a multiplicative process that models returns to investment in financial markets.

Europe and USA. Firm growth could be modeled as clustering process. Clustering of large number objects was provided by means of Zipf and Yule distributions [10]. Gibrat rule of proportionate growth claims that size of firm and its growth rate are independent. In many cases, Girbat law contains empiric error due to stochastic grown process [11].

Galeo et al. [12] analyse very large amount of data containing G7 group's firms over the period 1987÷2000 in several business cycle phases. Power law distributions are satisfied in all cases, but differences between parameters related to the recession and expansion processes are significant (the exponent $\gamma \to 1$, i.e., the resulting size distribution generally is not zipfian).

Axtell [13] analyses the distribution of USA firm sizes at historical perspective. Zipf distribution at lognormal scale takes place. Wyart et al. [14] studied Sutton 'microcanonical' model for the internal organization of firms as an alternative model based on power-law distribution. In that case, growth rates are asymptotically gaussian, whereas empirical results suggest that the kurtosis of the distribution increases with size.

Amaral et al. [15] analyse the Compustat data base comprising all publicly-traded United States manufacturing firms within the years 1974-1993. Amaral concludes the distribution of the logarithm of the growth rates, for a fixed growth period of one year, and for companies with approximately the same size, displays an exponential form.

Asia. Taking into account the parameter expressing the size of firm, power law expressions allow to receive the correlated distributions involving both processes: destruction and creation [16-17]. Some kinematical relationships between Pareto–Zipf and Gibrat laws are presented by Fujiwara [18]. Fujiwara et al. [16] analyse large number of European firms using power-law dependencies. Upper-tail of the distribution of firm size can be fitted with Zipf dependence, and that in this region the growth rate of each firm is independent of the firm's size. This sentence satisfied the Gibrat law.

Zhang et al. [19] analyse the data of top 500 Chinese firms from the year 2002 to 2007. Dependence of firm size on rank is presented according to Zipf law (exponent γ=1 for each year). Phenomenon explanation of it based on a simple economic model which takes capital accumulation into account.

Gupta et al. [20] studied the statistical distribution of firm size for USA publicly traded firms through the Zipf plot technique. Sale size is used to measure firm size. The log-normal distribution has to be gradually truncated after a certain critical value for USA firms. Therefore, the original hypothesis of proportional effect proposed by Gibrat is valid with some modification for very large firms.

Bankruptcy. Byoung Hee Hong et al. [21] studied the scaling behaviours for fluctuations of the number of Korean firms bankrupted in 2002-2003. Power law distribution of the number of the bankrupted firms takes place and Pareto exponent is close to unity.

Fujiwara [17] studied the data of Japanese bankruptcy in 1997. Zipf law dependencies could be estimated for the distribution of total liabilities of bankrupted firms in high debt range. The life-time of these bankrupted firms has exponential distribution in correlation with entry rate of new firms. Debt and size are highly correlated, so the Zipf law holds consistently with that for size distribution.

2. Wealth distribution

Souma [22] reported empirical studies on the personal income distribution, where two models were used: lognormal and power law. Pareto and Gibrat indexes were used as an unversal factors in order to estimate the temporal changes.

Europe and USA. Pareto distribution was devoted to describe the allocation of the wealth among individuals. In any society at any times the larger portion of the wealth (80% by Pareto [23], 70% by Gide [24]) is owned by a smaller percentage of the people (20% Pareto, 30% by Gide).

Hegyi [25] analyses the distribution of wealth in the medieval Hungarian aristocratic society. Wealth distribution was find according to power-law nature. Using no-trade limit of wealth-distribution model, Pareto law validity was confirmed for feudal society. Obtained Pareto exponent $\gamma \in [0.92 \div 0.95]$ is closed to 1.

Iglesias et al. [26] analyse the emergence of Pareto wealth power-law distribution. Models including the risk factor were proposed and tested. For constant risk aversion the system self-organizes in a distribution that goes to a Gaussian. Surprisingly, it was established that random risk aversion can produce distributions going from exponential to log-normal and power-law. Correlations between wealth and risk aversion was found.

Parameterization using temperature model occurs by solving unregular tasks of large scale wealth distribution. Dragulescu et al. [27] analyse the data on wealth and income distributions in the United Kingdom, as well as in several states of the USA. Great majority of population is described by an exponential distribution, and the high-end tail follows a power law. New empirical parameter – temperature (as analogy in physics) was introduced in order to characterize “the kinetic energy” of society.

Hernandez-Perez et al. [28] analyse company size distribution for developing countries using the framework proposed by Ramsden et al. [29]. Not adequate living conditions not allow to compare the usual makroeconomic parameters such as Zipf exponent etc. Hypothesis of additional parameter which plays a role analogous to the temperature of the economy occurs after decision that the level of economic development must be estimated in usual power-law dependencies.

Ausubel [30] analyses the myth *Living like America* from the economic perspectives. He claims from historical point of view that incomes vary for the very simple reason: income crowns the successful completion of a series of multiplicative tasks, causing a skewed distribution. As incomes rise, however, economic, social, and environmental requirements and capacities grow.

Trigaux [31] describes the main principles for tasks of econophysics and economy simulations. Ground idea could be formulated as follow: the repartition of wealth in every economic system always occur in Pareto law. As this inegalitarian repartition is a cause of many problems in the world, it would be interesting to find a remedy. Econophysics studies always start from the hypothesis as what economy systems are formed only of agents perfectly egocentric, each seeking only to gather the maximum of wealth for himself. Trigaux formulated two questions:

i) should the Pareto law come only of this limiting hypothesis (the maximum of wealth for himself)?
ii) should the Pareto law come in case if agents had other types of behaviours, for instance altruistic?

In order to simulate the proposed situation, two behaviours (altruism and egocentrism) were parameterized. Really much more egalitarian repartition appears, even with a relatively low rate of altruism (15%). More so, this egalitarian repartition occurs according to a Gauss law which is completely different law from that of Pareto.

Asia. Okuyama et al. [32] analysed the distribution functions of annual income of companies. Power-law distribution (according to Zipf law) was confirmed. Aoyama [33] analysed personal income, company’s income, and various measures of company size. Some relationships under the Pareto–Zipf law and Gibrat law of detailed balance were established as a basis for perturbative treatment of the economic change.

Isikawa et al. [34] analyse the database of high income companies in Japan. Quantitative relation between the average capital of the companies and the Pareto index was find. Quantitative relation between the lower bound of capital and the typical scale at which Pareto law breaks was established.

Theoretical study of the changes in poverty with respect to the ‘global’ mean and variance of the income distribution using Indian survey data was done by Chattopadhyay et al. [35]. Authors claim that Pareto poverty function satisfies all standard axioms of a poverty index presented by Kakwani [36] and Sen [37]:

i) monotonicity axiom: given other things, a reduction in income of a person below the poverty line must increase the poverty measure;
ii) transfer axiom: given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure.

Evolutionary games represent an important factor in simulating of economic environment. Mao-Bin Hu et al. [38] proposed full-time study of wealth distribution with agents playing evolutionary games on a scale-free social network. Pareto power-law distribution is satisfied for agent’s personal wealth prediction. Phenomenon of accumulated advantage (so called **Matthew** effect, the rich get richer and the poor get poorer) was validated also by analysing the agent’s personal wealth correlation to its number of contacts (connectivity).

3. Resourses and investment strategy

Naldi [39] studied the relationships between Zipf law and the major concentration indices. Standard model where the firms’ size are related to the financial investment amounts was used. It was established that Hirschman–Herfindahl index [40] is the most sensitive index in contexts where Zipf law applies. Applications of Zipf law could play an estimating role many very sensitive marked indicators.

Ausloos et al. [41] describe strategy how to apply the Zipf method to extract the γ-exponent for seven financial indices (DAX, FTSE, DJIA, NASDAQ, S&P500, Hang-Seng and Nikkei 225). Ausloos et al. [42] studied short-range time correlations in financial signals by means of Zipf method and the i-variability diagrams (VD). A precise Zipf diagram analysis has been shown to lead to a non-immediate information on the signal behaviour, even taking into account error bars.

Alegria et al. [43] probed to relate the parameters of Pareto-type distribution of bank sizes to the specific bank indexes such as Herfindahl–Hirschman index and the top 5%-concentration ratio. Effect of changes in Zipf exponent γ correlates to sample size. Wilhelm et al. [44] analyse an elementary stochastic model representing the system with finite resources where power-laws distribution takes place. This model extends the scale-free network model (SF) to include the fact of finite resources.

Saif et al. [45] investigate the problem of wealth distribution from the viewpoint of asset exchange. The simple asset exchange models (grounded on Pareto law) fail to reproduce trading strategies. Two models were used for successful simulation of trade:

i) Yardsale (YS) purpose model; and
ii) theft and fraud (TF) model.

Power-law tail in wealth distribution was observed in case if the agents are allowing to follow either of the mentioned models with some probability.

4. Trading / stock market models

The most important task is to create the dynamic market model which could predict the trade type and day-to-day fluctuations. Balakrishnan et al. [46] studied and modeled the distribution of daily stock trading using the power law. New phenomenon was established that the trading is becoming increasingly concentrated in a subset of stocks. The power law exponent systematically increases with time suggesting. Tuncay et al. [47] analyse the daily financial volume of transaction on the New York Stock Exchange and its day-to-day fluctuations. Gaussian distribution for longer time intervals, like months instead of days takes place. Otherwise, power-law tails could be attracted to long-term trends. Unconditional volatility distribution [48] of the Italian futures market were studied by Reno et al. Transactions in period of 2000 and 2001 (including event of dramatic 11 September 2001) were characterized by unusually high volatility levels. Results show that the standard assumption of lognormal unconditional volatility has to be rejected for such a turbulent sample, since it is unable to capture the tail behaviour of the distribution.

Gabaix et al. [49] presented a theory of excess stock market volatility. Market movements are due to trades by very large institutional investors in relatively illiquid markets. Power law distribution can be presented for resuming evaluation of trade, but optimal trading behaviours are stochastically-dependent.

Ideal-gas-like-models. Chatterjee et al. [50] reviewed big number of market models differing by shape of the distribution of wealth. Several paradigms from physics such as ideal-gas-like models of markets are observed across varied economies. Presented realistic model where the saving factor can vary over time (annealed savings) is yielding the Pareto distribution of wealth in certain cases. Numerical simulation presented in Ref. [51] describes the ideal-gas model of trading markets, where each agent is identified with a gas molecule and each trading as an elastic or money-conserving two-body collision. Unlike in the ideal gas, quenching/ saving properties are included. Model is showing self-organized criticality, and combines two distributions: Gibbs and Pareto.

Bhattacharyya et al. [52] obtained common mode of origin for the power laws:

i) the Pareto law was used for the distribution of money among the agents with random-saving propensities in an ideal gas-like market model; and
ii) the Gutenberg–Richter law for the distribution of overlaps in a fractal-overlap model for earthquakes.

Lotka–Volterra formalism. Market stability was studied using the generalized Lotka–Volterra (LV) formalism by Louzoun et al. [53]. LV equations are non-linear differential equations, pair of first-order, frequently used to describe the time-dependent dynamics of biological systems in which two species interact, one as a predator (y) and the other as prey (x):

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha x - \beta xy \tag{10}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \delta xy - \gamma y \tag{11}$$

Parameters α, β, γ and δ describe the interaction of the two species. First derivatives of x and y represent the growth/decreasing rates of the mentioned populations over time.

Power law distributions in the individual wealth (according to Pareto law) and financial markets returns (fluctuations) show auto-catalytic or multiplicative random character of the capital dynamics. Exponent of the power laws turns out to be independent on the time variations of the average. This explains also the stability over the past century of experimentally measured Pareto exponent. Strong feedback signalizes the danger of the market stability.

Solomon et al. [54] adapted generalized LV model with mutiagent systems in order to investigate economic systems. Weak generic assumptions on capital dynamics were realized in model of predictions for the distribution of social wealth. In ‘fair’ market, the wealth distribution among individual investors fulfils a power law.

Simulations and games. Chebotarev [55] propose the study of a hierarchical income model for asymmetrical transactions: directions of money movement and commodity movement are opposite. The price-invariance of transactions means that the probability of a pairwise interaction is a function of the ratio of incomes, which is independent of the price scale or absolute income level. The income distribution is a well-defined double-Pareto function, which possesses Pareto tails for the upper and lower incomes. The Pareto exponents are also stable with respect to the choice of a demand function within two classes of status-dependent behaviour of agents.

Mohanty [56] presented an economy model by taking N independent agents who gain from the market with a rate which depends on their current gain. Power-law distributions take place. Kuscsik et al. [57] studied the model of environmental–economic interactions. The interacting heterogeneous agents are simulated on the platform of the emission dynamics of cellular automaton. Steady-state and non-equilibrium properties were established in such type simulation. Relationship to Zipf law and models of self-organized criticality were discussed.

Yanagita et al. [58] studied a simple model of market share dynamics with rational consumers and firms interacting with each other. Simulation results show that three phases of market structure appear depending upon how rational consumers are. Three phases could be titled as the uniform share phase, the oligopolistic phase, and the monopolistic phase.

In an oligopolistic phase, the market share distribution of firms follows Zipf law and the growth-rate distribution of firms follows Gibrat law. An oligopolistic phase is the best state of market in terms of consumers' utility but brings the minimum profit to the firms because of severe competition based on the moderate rationality of consumers.

5. City creation mechanism

City growth phenomenon is well known from the Antic time as the parameter of civilization development. Growth is stimulated by the human activity in case if the resources are in enough amounts. City growth process could be described according to the power-law dependence as was checked in the middle of XX century. Formulated as an universally law, city size distribution was related to the power function. So called **Zipf law for cities** (exponent γ=1) was treated as the quit enough power law realization.

$$f(r) = \frac{\alpha}{r^1} \tag{12}$$

Nitsch [59] provides very large study of the empirical literature on Zipf law for cities including 515 estimates from 29 studies. Surprisingly, Zipf exponents are significantly larger than 1.0. This finding implies that cities are on average more evenly distributed than suggested by Zipf law.

Marsili et al. [60] presented a general approach to explain the Zipf law of city distribution. Benguigui et al. [61] presented an application of a growth model for a system of cities (computer model simulation). Model includes a random multiplicative process for the growth of individual entities and for the creation of new ones. Expression with a positive exponent -"shape exponent" and additional three parameters was used in order to describe the dynamics of the systems' size distributions through time. Quit good agreement at the macro level between the model and the real data takes place.

Pareto distribution allows to make the very strong city size estimation in many countries. Soo [62] solved the task of empirical validity of Zipf law for cities, using data on 73 countries. Two estimation methods - OLS (ordinary least squares) and the Hill estimator – were used. The OLS estimates of the Pareto exponent are roughly normally distributed, but those of the Hill estimator are bimodal. Variations in the value of the Pareto exponent are better explained by political economy variables than by economic geography variables. Cordoba [63] derived the conditions in the framework of Pareto model. Presented rules must satisfy the standard urban model:

i) a balanced growth path; and
ii) a Pareto distribution for the underlying source of randomness.

Gabaix [64] presented review surveys of well-documented empirical power law regarding income, wealth, the size of cities etc. Random growth, condition optimization must be treated as the adjustable parameters. Some empirical regularities currently lack an appropriate explanation. Gabaix also describes the open areas for future research.

City size represent a geometrical distribution of urbanized areas. Benguigui et al. [65] presented the growth model for a system of cities which is grounded by not only Zipf law but also other kinds of city size distributions. Power-law like function with exponent γ (for Zipf law γ=1) was introduced. Three classes of city size distributions depending on the value of γ were defined: i) γ>1; ii) γ<1; iii) γ=1. The model is based on a random growth of the city population together with the variation of the number of cities in the system. It was concluded that the exponent γ may be larger, smaller or equal to 1, just like in real systems of cities, depending on the rate of creation of new cities and the time elapsed during the growth. It is necessary to point out that the influence of the time on the type of the geometric distribution must be treated as significance.

Carvalho et al. [66] studied the distribution of the length of open space linear segments, derived from maps of 36 cities in 14 different countries. By scaling the Zipf plot of l, two master curves for a sample of cities, which are not a function of city size, were obtained. It means that third class of cities is obtained, and this class is out of classification order. According to Zipf plot, this distribution is realized in region of power-law tails with exponent γ=2. Small correlation between real data and the possibility of observing and modeling urban geometric structures was suggested. Volchenkov et al. [67] studied the distribution of open space in city. The area of open space which are related to the other spaces is distributed according to the power-law statistic. Observed universality may help to establish the international definition of a city as a specific land use pattern.

Stochastic model of city growth represent a behaviour of cluster formation type where time-dependent processes occurs. Zanette et al. [68] proposed stochastic model for govern city formation. The model predicts a power-law population distribution whose exponent is in excellent agreement with the universal exponent observed in real human demography. Zanette suggested that urban development at large scales could be driven by intermittency processes. Duranton [69] presented canonical model of endogenous growth with product proliferation into a simple urban framework (which yields Zipf law for cities). The stochastic outcomes of purposeful innovation and local spillovers can thus serve as foundations for random growth models.

6. Driving forces for city expanding

Mansury et al. [70] presented a spatial agent-based model to generate a system of cities that exhibits the statistical properties of the Zipf Law. Two main factors could be estimated as of most important significance: bounded rationality and maximum heterogeneity of agents. Combination of such two factors can produce a generic power law relationship in the size distribution of cities, but does not always generate the dependency according to Zipf law. Zipf law breaks down unless the extent of agglomeration economies overwhelms the negative disagglomerating forces. Decker [71] probed to solve the city growth task when largest cities comprise the long tail of the distribution. In order to explore generating processes, simple model was used. Model incorporates only two basic human dynamics: migration and reproduction.

Semboloni et al. [72] presented the model for the distribution of individuals in cities. The number of individuals is fixed and the dynamic depends on migration from one city to another. Two strategies were used for modeling purposes: utilisation of resources for production and selling of products to people. The most important statements can be formulated as follows.

1. Because resources are uniformly distributed and shared among individuals, the first strategy pushes individuals in small cities - *unification*.
2. In turn, because selling depends on the quantity of individuals are living in a city, the second strategy pushes individuals in big cities - *diversification*.

Random application of unification and diversification strategies results in power-law distribution of cities.

Europe. Sarabia et al. [73] introduced the Pareto-positive stable distribution as a new model for describing city size data in a country. The mentioned distribution provides a flexible model for fitting the entire range of a set of city size data. The classical Pareto and Zipf distributions are included as a particular case. City size data for Spain for several different years was considered. The new distribution is compared with three classical models: Pareto, lognormal and Tsallis distributions.

Asia. Anderson et al. [74] analyse city size distribution in China using two behaviours: i) the relative growth of cities and ii) the nature of the city size distribution. This analysis was provided in the framework of political conditions such as Economic Reforms and the One Child Policy since 1979. It was established as a reason for the significant structural changes in the Chinese urban system. The city size distribution remains stable before the reforms but exhibits a convergent growth pattern in the post-reform period. It was concluded that log-normal rather than Pareto specification turns out to be the preferred distribution.

Gangopadhyay et al. [75] studied the size distributions of urban agglomerations for India and China. Authors have estimated the scaling exponent for Zipf law with the Indian data (1981-2001) and Chinese data (1990-2000). Parameters of Pareto and Tsallis q-exponential distribution have been estimated: for India, $\gamma \in [1.88 \div 2.06]$ and for China, $\gamma \in [1.82 \div 2.29]$.

Chen [76] examined the relation between the feature of increasing returns in the dynamic growth process and the property of power law in the static limiting distribution. Fractal-like structures used in this model implies both the power law and rank size rule. Power law or Zipf law are valid for the distributions of city size. Gibrat law proposes general and neat interpretations for this regularity in a city distribution, but the homogeneity assumption in Gibrat law shows a disregard of the agglomeration effect that is essential in economic interpretation. Path-dependent nonlinear Polya processes were appended to analyse the relation between the feature of agglomeration in the path-dependent processes and rank-size relations in the limiting distributions. Author conclude that the assumption of agglomeration economies must be significant. It allows to state that the agglomeration benefits increase without a ceiling as the residents are added to the city.

South America. Moura et al. [77] studied the application of Zipf law for cities distribution in Brazil. The results show that the population distribution in Brazilian cities does follow a power-law similar to the ones found in other countries. Values of the power-law exponent were found to be about $[2.2 \div 2.3]$. More accurate results were obtained with the maximum likelihood estimator, showing an exponent equal to 2.41 for 1970 and 2.36 for $2003 \div 2006$.

Conclusions

1. Power function dependence in Zipf law realization allows to conclude that popular regularities in economics (zipfian and also logarithmic) can have the common stochastic origin.
2. Zipfian behaviour is encountered also in chaotic dynamical systems with multiple agents (attractors). Deviations from linear dependencies in log-log scale allows to state the presence of perturbations.
3. Time-dependent or parameter-dependent exponent dynamics (from Zipf-Mandelbrot and Yule dependencies) allow to model and to estimate the survivor of economic system (self-organised criticality).
4. Cities are on average more evenly distributed than suggested by Zipf law.

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