Spatial self-arrangement of expanding structures. 2. Structure model construction

Jelena Nesterova [a]
Department of Geophysical Sciences, Faculty of Mathematics and Natural Sciences,
Klaipėda University, H. Manto st. 84, LT-92294 Klaipėda, Lithuania

Received 14 April 2010, in revised form 2 September 2010, accepted 5 November 2010

Abstract. Vital systems distinguish in some typical features making them resistant to disturbing action. Their ability to take advantage from interaction with surrounding is called *self-organization* (pointing the functions) or *self-arrangement* (considering the spatial traits). Research of such organized systems (because of their affinity to life and human race) outspread globally. Theories of non-linear and non-equilibrium dynamics, of chaos and dissipative structures, the fractal geometry and other branches of modern science have been induced by organization problems. Spatial features typical for most of organized structures and the quantification problem of this property are discussed here. Two measures of spatial organization dissimilar essentially in their nature have been studied more in detail. One of them arises from the perception of information; other is deduced from dynamical equations. Review of structure model construction is reported in this study.

Citations: Jelena Nesterova. Spatial self-arrangement of expanding structures. 2. Structure model construction – *Innovative Infotechnologies for Science, Business and Education,* ISSN 2029-1035 – **2(9)** 2010 – Pp. 23-29.

Keywords: Spatial organization; Expanding Structures; Organized structures; Indices of organization; Informational organization; Self-arrangement; Metrical organization; Cartographic structures; Areal; Genome; Morphome; Binome.
PACS: 91.10.Da
Short title: Structure model construction - 2.

Introduction

Populations, settlements and other real structures are too complex for research of their organizational features. Best what is possible here is to try to exploit some assorted models of these structures. So the modeling is the stage from which depend the success of guiding research. The robust model should meet some requirements of quality. First, it should be as simple as possible (since the simplicity conditions the efficiency of mathematical analysis). Second, it should keep the essential features of reality (it must be *complete*). In order to identify these properties (simplicity and completeness) it is necessary to extend the viewpoint from a structure of research to the whole reality (herewith entering the philosophy sphere) [1]. Such a generalization helps to stay within bound of reasonable simplification of a model sustaining herewith its minimal similarity to the reality.

Organized structures may be described as a type of spatial order, also known as *negentropy*. Previous publication represents an overview of paradigms [2] used for so complicated modeling. One type of indicators of the organization are derived from the models of general dynamic processes, while others show specifications of a single process, thus making them either too general (reflecting rather theoretical than practical point of view) or too narrow.

Purpose of this work is to look more comprehensively at this construction with aim to employ a model suitable for the prospective study of cartographical images which allows to describe two-dimensional distribution of real structures - forest areas using several modern assumptions such as fractal tools.

1. Model description

As it was pointed above, the simplicity and similarity to reality are two greatest advantages of a model. The amount of meaning kept in the simplicity would be proportional to knowledge and understanding of reality applied to a model. The requirement of simplicity is also valid for axioms, for example, a model gets better when the amount of axioms and principles used to describe it is lesser. According to objectives and the object itself, other properties also may be signi-

[a]email: *jelena.nesterova@gmail.com*

ficant. The most significant principle for studying organized structures is the systemic principle that requires to model every structure with properties of structures of adjacent higher and lower scale (note the *ternary* composition by means of the principle: given-higher-lower). Let's consider a territorial view of population and its cartographic layer. It would be impossible to analyse such a model using effective (mathematical) methods. To overcome this limitation, a graphical structure model has to be transformed to some quantitative expression called a *mathematical analogue* of given model.

Study of systems by a logical scheme "*what would happen, if...*", that is, implication, is barely applied at all. Known logical models are based either on the uniqueness of an original territory, or on general traits of many territories, such as the arid region, but there are no models that would be based on both types. Ordinary notions of *simulation* and *modeling* are pointing difference of these approaches.

For example, territorial objects, such as Lithuania (state), Scandinavia (geographic peninsula), Raigardas (valley in south Lithuania), Curonian Spit (peninsula in west Lithuania) are unique in their spatial organization and history. Though one may find among them many similarities in renewal, reproduction, irreversibility, critical modes, conversions and (or) other dynamical properties. Consequently in order to study how such system manages to select the features suitable for its survival and development, it would be reasonable to apply the *simulation* procedure (relying on the individual properties). Respectively the *modeling* (vs *simulation*) would be kind of the preferable procedure (perhaps...) for research of their dynamics (relaying on that what is common - emphasizing the similarities).

Now let's assume we face a change of situation, when the structures become different in dynamical, though similar in spatial features. How such a change should be reflected in construction of a structural model? What features should persist in a model when the structure is passing from one to another situation? There are no compelling answers to the later question. Some attempt to approach them is proposed further.

1.1. Philosophy of modeling

Theoretical structures are often compared with stained glass, carpet or a puzzle, although, it does not reveal the point and content of landscape. The whole idea of landscape should be

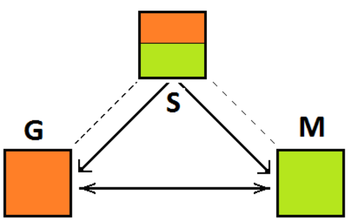

Fig. 1. Structure as an ontological triplet:
G – genome, M – morphome, S – symphysis.

pervaded with features of dynamics, self-organization, vitality. This requires the specific standpoint, or, to be more exact, return to one of forgotten viewpoint [1].

According to Raicinskis [3], Kant [4], philosophy of Dao [5], every object is triple inside. Each of them is constituted of three inherent origins. Separate compounds could be described as following:

a) active origin, *genome*,
b) passive origin, *morphome*,
c) substance receiving after allowing first two origins act together, *symphysis*.

Genome is usually accounted for cause, morphome – for shape, symphysis – for bonding between genome and morphome (see Fig. 1).

Symphysis may be treated as an informational background of reality. Sometimes it is used to describe the existential potencies of a structure. Several examples are presented below:

i) energy-information-their junction;
ii) past-present-future;
iii) matter-antimatter-vacuum;
iv) positive unit-negative unit–zero;
v) something-nothing-anti-something.

There are well known interpretations from physics: from nothing may not appear something, however from nothing may appear and exist something and anti-something. All of them are also the possible interpretations of this general principle. It is natural that a particular basic element is difficult to identify, as the reality appears from "association of three". And again the structures are composed of different sort of substructures and their boundaries are quite vague (see Fig. 1). So it is more credible for actual model that basic elements will be assumed with some probability to satisfy their definitions than identified certainly. This shortage does not debase the principle itself: a testable mistake is better than the total defiance of the ontological nature of structure [1].

Symphysis describes the bond as well as the possibility, existential potencies of a structure, area of geographical structure. Symphysis is an information model of reality. Active and passive elements of evolving structure may not be defined and constant as well. They are made of difficult inner structure and different origin.

1.2. Graphical scheme

Now the structure model may be displayed in geometrical form as a trigram [1] - see Fig. 2. Real structures have geometrical features. The trigram is lacking them, sustaining only the topological mean. The symphysis is represented here by a spatial niche called *areal*. It always stays as a background for other two spatial elements (patches). A patch representing the genome commonly is put in a morphome (representing a functional space) though sometimes both patches can be spatially separated or dislocated in adjacent position (Fig. 3). In some other cases it is reasonable

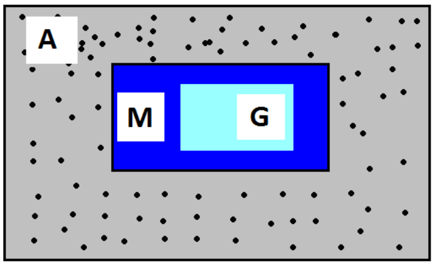

Fig. 2. Structure according to ontological framework.
G – genome, M – morphome, A – areal as symphysis.

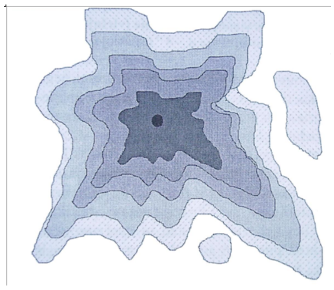

Fig. 3. Map of areal as twodimensional distribution.
Each isoline describes the different areal.

to reduce the genomes to points or to represent them as patches coinciding with their functional zones (morphomes).

1.3. Areal

Definition and contour of areal returns the axiom to a model graphically in a way where every specific phenomenon of evolution exists in a specific, real and finite spatial area. If evolution of a structure depends only on properties of inner environment, then areal is not a part of a model of discussed structure. If latter condition is fulfilled, areal characteristics are excluded from the equations that describe the structure. Although, if it is excluded, it may miss some hidden parameters, so it is more convenient to use two definitions of areal. Potential areal becomes part of a developing system once it starts interacting with it. The ability of interaction is defined by a probability. Once the interaction between a structure and environment begins, their evolution becomes combined and areal becomes as a part of a structure [1].

The environment of developing structure is made up of interacting elements. Changes that occur at the elements of environment force a change in areal as well - its characteristics, such as size, form, topology, may vary. Although topological dimension of a curve is equal to 1 but otherwise general dimension of a figure is equal to 2. That is a reason for a new type of dimensions - Hausdorff dimension D_H.

Areal is very difficult and constantly changing structure, although it is usually considered as a constant area or contour as presented in Fig. 3.

It is difficult to determine boundaries of areal while it is almost impossible to forecast the changes of areal because structure observed is only one of many structures and independent factors that affect and force changes in it. In order to obtain more accurate result in geographic studies it is necessary to find solutions that are independent, or depend on characteristics of an areal the least as presented in Fig. 4.

If model parameters of areal are more important than the risk of failure, boundaries of areal are modeled according to a specific procedure, for example methods of Tiessen and Voronoj assume that part of areal is proportional to measurements of objects or their distances of interactions. All the area is divided to smaller parts in a way that their boundaries would be of similar distances to nearby binomes.

Such division of areas may look artificial at first sight, but it is not. If there are no visible boundaries in reality, it does not deny their existence. Many of boundaries drawn in cartographic models actually exist in reality. There also are boundaries that exist, but nothing is known about their location, origin and properties. Median boundaries summarize, average past and present, real and imaginary, actual boundaries by reflecting the most common properties. Boundaries of morphomes are modeled alike in cases when genome is an island on morphome. Morphome in such case is like a microareal of genome.

1.4. Genome and morphome

Pairs of these elements allow modeling most of the surrounding world. Every element of binome belongs to parts of structure that are essential for the other element, which, in accordance is essential for existence of functioning and development of the structure itself. Every element in binome has its own geometrical and topological properties and changes of it. The proportion between elements is equal to (1/N).

For example, if the genome represents rural area (objects of one cartographic layer), morphome represents many layers: land, forests, waters. Sometimes, while creating a model it is unclear which element is active and which is passive - proportion between the elements of observed structure is unclear. In that case, simpler models, such as cartographic views and their graphic schemes may become handy [1]. Usually, *genome* is considered as an element that is capable to develop by itself, while *morphome* is a complimentary

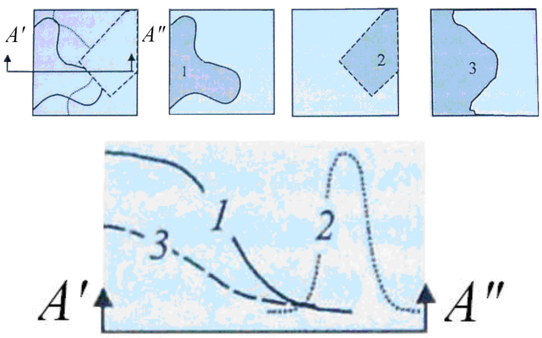

Fig. 4. Area parts of three different areals (top). Density of corresponding components in the cut A' – A'' (bottom)

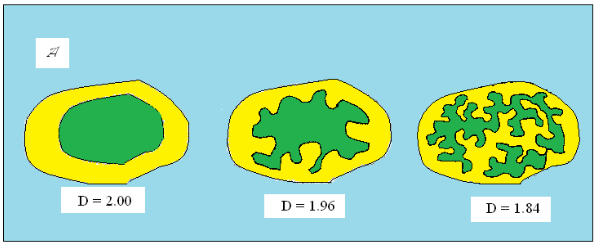

Fig. 5. Different forms of islands after change of fractal dimensionality. D – fractal dimension.

element for genome. For example, if we would consider population an active element, it would be appropriate to consider its habitat as a passive element of a structure. Interpretation like this is quite frequent, but it is optional. In reality, structures and parts of them vary by their geometrical specifications and change of them. Structures also should differ by their size, form and binome element. Binome may expand to the size of areal (A), but may never exceed it. Dots and lines in a model represent small and elongated elements that may never become equal to zero.

In our reality there exist more difficult alterations. In order to model them, islands should be transformed to a background or islands may be formed in other islands, gaps that do not belong to areal may appear on the areal. For example elements of binome may be different by topological proportion: in one case, by removing one of elements, the other element stays the same. In other case, it shrinks and gap appears in its place. Point elements may become linear, while the latter may become an area. That means, that elements may change their dimension - see Fig. 5.

1.5. Binome

Territory that is observed may be divided to an infinite number of smaller parts and defined by trinomials. It is obvious that these models will belong to different types because they represent different objects, such as atoms and ecosystems [6].

The same territory may be divided to parts that have analogic meaning or origin, so the amount of them would be significantly smaller. These fragments still have some common properties but may lose them, so it is impossible to further divide them to analogical type as they are themselves. Fragments like these are called elementary, their graphic models are binary, models of binary elements - spots. Elements in such model have geometrical and topological meaning.

Every evolving structure can be modeled according to the fact, that they are open and binary - every structure has an active element, genome, and part of environment that genome interacts with and that is the second side of the same area. Passive element may be referred to a functional area or to the area of interaction. Otherwise, elements form the random crossing.

The difference between traditional and proposed model may be compared to difference between a community and an ecosystem: landscape acts as a community in a cartographic model. Binary model displays community with its habitat, by highlighting its structure, features and spatial bonds. Binary model is appropriate for modeling of non-reproductive and interacting structures, such as rivers and their basins.

There are two ways to convert traditional single-member model to binary model. That is achieved by converting single-member component as a passive or an active component of binary model. When active component shrinks and becomes a singular point, binary model degenerates and becomes a single-member model, so traditional representation of components on a map is a partial case of a binary model. Every evolving structure may be put together or divided to binary components and their layers. They can be easily transformed to elementary quantitative expressions and studied by using effective mathematical methods. This ability is the most valuable feature of the method. Of course, this method, like any other has its drawbacks, for example, limits of functional space have to be calculated in this method, but these drawbacks do not outshine the merit of this method.

2. Indexes of metric organization

Evolution of population of organisms and similar structures depends on their geometric shape. This property (metric organization) is defined by an indicator M_i - see Refs.[2,7].

2.1. Fractal dimension

The indicator of metrical organization is based on mathematical modeling of structures as well as describing the structure itself. One of such properties is fractal dimension. It is now known that it differs from topological which may obtain only integer values (0, 1, 2,...).

The term *fractal* was introduced by mathematician Mandelbrot in 1980's [lot. *fractus* > *fragere* - to break, to create an irregular ornaments].

Fractal structures, such as Cantor set, Pean curve, Koch curve, Sierpinski triangle and carpet, Julia set, also known as so called *mathematical monsters*, had been known before the introduction by Mandelbrot. Although, only Mandelbrot saw something more than mathematician's mind in them and founded new branch of mathematics - *fractal geometry*. In less than 20 years term of fractal found its place in other sciences: physics, biology, chemistry, sociology, urbanistics. As it has already been mentioned - attractors of chaotic systems are fractals. That's why fractal geometry is actually *chaos geometry*.

Fractal is object that has a property of self similarity - object may be divided to small parts that look like shrunken reflection of a structure, for example, branch of a tree looks like a shrunken tree, that's why we can call trees as natural fractals. There are a lot of natural fractals in the nature - from plants and human organs(kidney, lungs) to mountains,

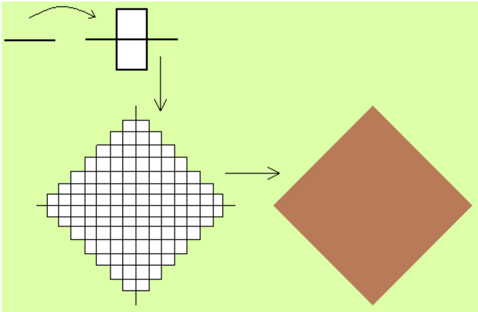

Fig. 6. Pean curve. First three steps of formation and final result - totally filled surface.

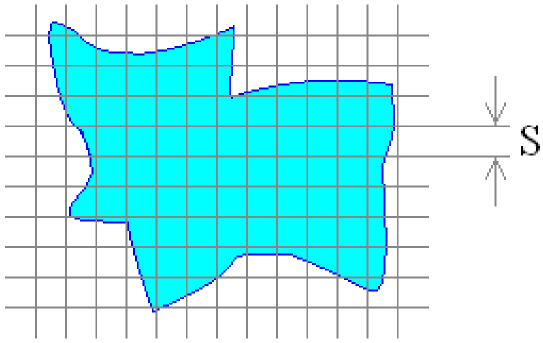

Fig. 7. Calculation of dimension of coverage.

clouds and shore lines. Some structures are only partly self similar, for example, trajectory of Brown particle is fractal, but it is only partly self similar. In such case fractal is an object where level of details increases while magnifying it. Mathematically, fractal is a structure whose Hausdorff dimension D_H is higher than topological dimension D_T:

$$D_H > D_T \tag{1}$$

Topological dimension D_T is classical assignation of dimensions of figures - it assigns 0 dimension for a point, 1 to a curve, 2 for a plane, 3 for a cube, etc. There may be no fractional dimensions. Later on appeared geometrical structures that had higher dimension than topological, for example, plane filling curve. Fig. 6. represents Pean curve as a typical example of plane filling curve.

2.2. Dimension of self similarity and coverage

The composition of most real structures manifest signs of *self similarity*, D_S. This feature can be measured by following expression:

$$D_S = \frac{\log a}{\log(s^{-1})} \tag{2}$$

where a - partition number, amount of parts the figure may be divided to; s - factor of magnification, the number of times of magnification in order to get a part same sized as a initial figure. As an example, dimension of a square may be calculated: square can be divided to 4 smaller squares.

After mentioned operation half-sized copies of the figure itself occur. As we see - fractal dimension of simple figures is equal to their topological dimension.

Fig. 7 represents the calculation scheme of *coverage* dimension. A figure is covered by squares - in case if it is on a plane. In the case of n-dimensions, it is covered with n-dimensional or n-D cubes. The length of an edge is equal to S_k. The amount of squares that take up the figure, N_k is calculated. Computations are compared with length of an edge of another square $S_{k+1} < S_k$. Then, the dimension of coverage is presented below:

$$D_b = \frac{\log N_{k+1} - \log N_k}{\log(\frac{1}{s_{k+1}}) - \log(\frac{1}{s_k})} \tag{3}$$

If it is covered with any other elementary n-dimension units of volume and the radius of those volume units approaches 0, the dimension of coverage D_b becomes Hausdorff dimension D_H:

$$D_H = \lim_{s \to 0} \frac{\log N(s)}{\log \frac{1}{s}} \tag{4}$$

2.3. Power law to fractal structure

Another way to describe fractal dimension can be used for practical purposes. Lets suppose our measured unit is u. If s^{-1} is the accuracy of measurement, then the power law may be applied to fractal structure:

$$u \approx s^{-d} \tag{5}$$

As we set the logarithm of accuracy $\log_e 1/s$ on x axis and the logarithm of measured unit $\log_e u$ on y axis, we get a straight line whose slope is d. Fractal dimension $D_C = d+1$.

Using this method, the dimension of Great Britain island shore had been measured: $D_C = 1.31$. Actually, as we measure with rulers of different accuracy, we get different length of a shore line. That is the case, because, as we increase our accuracy of measurement, we take even smaller abruptness into consideration.

Table 1 represents the typical fractal objects with the parameters. Fig. 8 represents Koch fractal (left) and Pean fractal (right). Fig. 9 represents Sierpinski triangle (left) and trace of Brown particle (right).

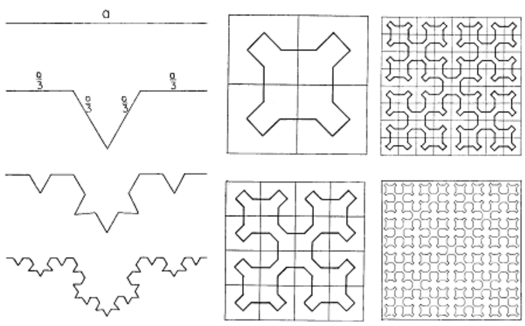

Fig. 8. Koch fractals (left) and Pean fractals (center, right).

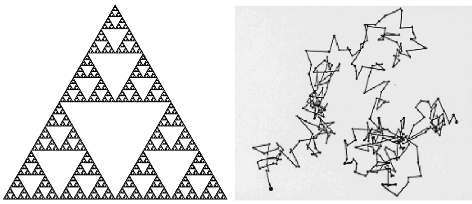

Fig. 9. Sierpinski triangle (left) and trace of Brown particle (right).

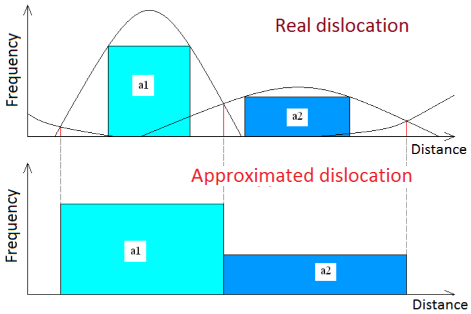

Fig. 10. Scheme of spatial bordering of individs.

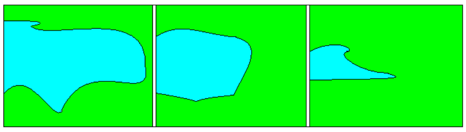

Fig. 11. Surface areas used for modeling.

Besides these three fractal dimensions there are more - information, entropy, etc. Usually, various dimensions on the same object are equal, but sometimes they might differ.

3. Possible description of earth surface layers

In order to calculate the indicator of metrical organization, a function of observed objects has to be specified, thus forcing us to specify a model. The calculation of the indicator of metrical organization is based on forms of spatial units in order to determine the difference in organization of a structure after the last crisis [7].

Table 1. Typical fractal objects.

Object	Topological dimension	Fractal dimension
Curve	1	1
Square	2	2
Cube	3	3
Koch fractal	1	1.26
Pean fractal	1	2
Sierpinski carpet	1	1.89
Brown movement trace	1	2

Actually, elements should be displayed as in top of Fig. 10, because forests are live and evolving structures. When creating a model, we have to predict that the structure is likely to grow or shrink. Although, we consider that it will remain constant when making a model - see bottom of Fig. 10.

Also, we need to know that areal may change - grow or shrink, although, it is defined as a H=const in this model.

The best example of changes of habitat is dynamics of tides. Three states of area of land are represented in Fig. 11: first one defines a high tide - areal is significantly bigger than areal in second state, that is neutral. Third one is low tide - area of areal is significantly reduced [8-9].

Conclusions

1. The simplest part of a structure sustaining yet the essential properties of reality is the ternary system. Three basic elements at least should present (in recognizable form) in any quantitative model of a real structure.
2. Binary and unitary formalism is incomplete and insufficient to reflect the nature and essence of real developing systems.
3. To recognize the basic elements of a ternary model each of them should be treated as a ternary system.
4. Fractal description is able to reveal and describe the typical features of different structures.

Acknowledgments

This work was prepared as a part of the Master Thesis. Author would like to thank Assoc. Prof. Algimantas Tiknius (Klaipėda university) for suggestions and comments.

References

1. Algimantas Tiknius. Territorial Aspects of Development. Monograph (in lith.) - Klaipėdos Universitetas, 2002.
2. Jelena Nesterova. Spatial self-arrangement of expanding structures. 1. Overview of assessment concepts – *Innovative Infotechnologies for Science, Business and Education,* ISSN 2029-1035 – **2(9)** 2010 – Pp. 17-22.
3. Raičinskis K. K. Searches and Findings (in lith.) Vol. 1. – Austėja, 1992.
4. Watkins E. The Argumentative Structure of Kant's Metaphysical Foundations of Natural Science. – *Journal of the History of Philosophy* 36(1998) 567-593.
5. See textbooks devoted for Dao phylosophy, in particular case: Welch H., Seidel A. Facets of Taoism. – Yale University Press, 1979. – 147 - 151.
6. Ovaskainen O., Hanski I. Spatially structured metapopulation models: global and local assessment of metapopulation capacity. – *Theor. Popul. Biol.* 60(2001) 281–302.

7. Tiknius A., Jankauskas B. and Juška V. A method for Quality Assessment of Environment at the Landscape Level. – In: F. Muller, W. Kepner & K. Caesar (eds.). EcoSys 2003, Landscape Sciences for Environmental Assessment, Proceedings from a CCMS Pilot Study Meeting at Salzau Castle, Kiel, Germany, 2003. ISSN:0940-7782 – 56-70.
8. Tiknius A., Jankauskas B., Some effects of population growth on its spatial structure. – In: EcoSys, Landscape Sciences for Environmental Assessment, Proceedings from a CCMS Pilot Study Meeting at Lecce, Italy, September 6-9, 2004 – pp. 7.
9. Cosma Rohilla Shalizi ir Kristina Lisa Shalizi: Quantifying Self-Organization in Cyclic Cellular Automata. – <http://www.cscs.umich.edu/ crshalizi/research/FN03.pdf>, accessed 2010 06 12.