Spatial self-arrangement of expanding structures. 1. Overview of assessment concepts

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Abstract. Vital systems distinguish in some typical features making them resistant to disturbing action. Their ability to take advantage of interactions with surrounding is called *self-organization* (pointing the functional aspects) or *self-arrangement* (considering the spatial traits). The research of such organized systems (due to of their affinity to life and human race) has outspread globally. Theories of non-linear and non-equilibrium dynamics, of chaos and dissipative structures, the fractal geometry and other branches of modern science have been induced by organization problems. Present work is but a small touch to this great topic. The spatial features typical for most organized structures and the quantification problem of this property have been discussed here. Two measures of spatial organization dissimilar essentially in their nature have been studied more detailed. One of them comes from the perception of information; other is deduced from dynamical equations. Review of organization assessment concepts is reported in this study. Main paradigms - system, structure, information - and corresponding parameters - entropy, negentropy - are described for characterization two different - metric as well as information system.

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Introduction

Generally, populations, settlements and other developing structures are possessed by features sustaining their existence, survival or prevalence. It is significant to study these features – to learn to recognize and measure them as promising new ways of indirect (simpler and more effective) management of development phenomena. The main attention of this paper is focused on the spatial organization – to (geo)metrical and topological features giving the functional advantages to the structure.

1. Task formulation

The informational measure R of spatial organization is grounded on the Shannon entropy H. This function can be defined as the distribution of spatial features of the individuals at fixed moment of time (i.e. making untold assumption of ergodicy of phenomenon). Similarly the metrical measure of spatial organization M can be defined on the statistical features of spatial units. It is derived from dynamical equations and therefore it is carrying terms imperative for development [1]. One may note some links between these measures. Really, probabilities of spatial states pi, governing values of entropy H and values of measure R, depend on distribution of metrics (x,y, ...). Statistical features of individuals (comprising the given distribution) determine values of the metrical measure M. So it is reasonable to expect that values R_i and S_i (calculated for the same structures) should be significantly correlated. Otherwise, each of them reflects different properties of spatial organization. In order to test this assumption, comparative study of both measures should be performed using the common and real data.

2. Historical overview

The term *cybernetics* is known from Ancient times in the context of “the study of self-governance”. Science about the ruling of environment or nation was called cybernetics

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by Pluto (428-347 B.C.) [greek *kybernētēs* - steersman, governor]. According to scientific notation of the end of XX century, interdisciplinary study of the structure of regulatory systems was defined by Norbert Wiener.

French physicist A. M. Amper (1775 - 1836) also observed cybernetics as a form of science. A famous russian medic, philosopher and scientist A. A. Bogdanov - Malinovskij (1873 - 1928) published an original philosophical treatise [2] that represents the general idea of cybernetics - theory of organized systems. Unfortunately, the science and culture were not ready for publications of Pluto, A. M. Amper and A. A. Bogdanov at that time.

Clause Shenon (1948) in pioneering work [3] proposed the measurement of something new - *information*. His work was treated as a fundamental work for the theory of information that constituted science of cybernetics. The new concept of information was a standpoint of mathematical communication theory [4,5]. The main difference between cybernetics and physics or chemistry is the way the system works - opposed to the way of physics and chemistry, it does not move along the path of the highest probability. According to notation of the founder of modern cybernetics - Ross Ashby, cybernetic system selects the reaction pathway on direction not related to the highest probability. Cybernetic system performs the different actions using the information received. These systems were called *cybernetic* as well as *organized*.

Physicists, as well as various analysers of system describe their object of focus - *system* - as a function of cause-consequence and describe it as a mathematical - logical bond. In physics and chemistry, this is put together as a conversion between energy and material, characterised by transformation laws, while it is described as transfer functions that obey to general material laws, such as conservation law in the theory of systems and signals. It is known that tangible systems naturally move along the direction of the highest energy state. That is the second law of thermodynamics, also known as principle of Carnot that could be described as follows: "as time passes everything collapses, dissipates and all the differences and gradients become void". It is a parameter of unorganized physical-chemical structures [6].

Live forms of organisms, social-economical and even some of human-made technical systems that comply with law of energy conservation while do not comply with the second law of thermodynamics - they tend to move along the directions of even the smallest energetic states, thus causing the growth of gradients. Characteristic example is well known as the growth of embryo. Sniadecky [7] proposed the term *organized form* or *organic life form* in 1804. Nowadays, they are known as *organized systems*.

While observing various systems, we obtain data about that system (areas and numbers of our observed objects) information about them. When observing any system - whether it is organized or not, we can obtain generated sequences of data, making the system a source of information. Prediction of that kind enables observing the system from theoretical point of view. Information theory statistically describes properties of a signal instead of the ways the signal is transmitted. In other words, it describes the source of a signal.

Shannon introduced some interesting ideas and techniques to analyze sequences of discrete (binary) numbers generated by various systems. The main purpose of information theory was to analyze electrical signals but in a matter of several years it became indistinctable part of statistics, applied statistics, computer sciences, cryptography, biology and physics.

3. Description of system and structure

System. System [greek *systēma* – composition] is a set of interacting or interdependent components forming an integrated whole. It is not required that an element would be bond with every other element of the system, but an element has to have a bond with at least one element in the system. The properties of a system are described as a form of integrity between bonds of elements in the system.

Structure. Structure [lot. *structura* – framework, constitution] is a fundamental description that describes location and bonds of elements that make up any object. Structure is the essential property that describes the stability and quality of a system. Every structure has its own conditions of appearance (causes), and it interacts with other structures (consequences / effects), also affecting the first structure. Nevertheless, is should be considered that every structure is independent. Only then observing of structures becomes possible.

Entropy is a thermodynamic property that can be used to determine the energy of certain system available for useful work in a thermodynamic process. The term *entropy* was compiled in 1865 by Rudolf Clausius [greek *entropía, en + tropē* - in conversion.] There are two related definitions: thermodynamic and statistical mechanics. Thermodynamic entropy is a non-conserved state function. Increases in entropy correspond to irreversible changes in a system, because some energy is expended as waste heat. In statistical mechanics, entropy is a measure of the number of ways in which a system may be arranged, often taken to be a measure of "disorder".

Boltzmann entropy. The expression of entropy S used in thermodynamics and statistical mechanics [6] is Boltzmann entropy (initiators are Ludwig Boltzmann and J. Willard Gibbs, 1870):

$$S = -k_B \cdot \sum_{i=1}^{N} P_i \cdot \log_e(P_i) \qquad (1)$$

The summation is to be provided over all the possible states N of the system, and P_i is the probability that the system is in the i-th state.

Entropy in physics is a state function of a thermodynamical system that describes irreversibility of processes in an isolated system:

$$dS \geq \frac{dQ}{T} \quad (2)$$

where T - absolute temperature of a system, dQ - heat. For isolated systems dS=0.

Although this law does not apply to open systems, so, entropy describes very important class of processes - development processes (from physical to social).

Apparently, both sides of equation are equal for reversible processes and unequal for irreversible processes. The second law of thermodynamics is an expression of the system tendency that over time, differences in temperature equilibrate in an *isolated* physical system. In classical thermodynamics, the second law is a basic postulate applicable to any system involving measurable heat transfer and defines the concept of thermodynamic entropy dS.

4. Information

The term *information* means the knowledge that reduces or cancels uncertainty of occurrence of an event from possible events sequence [8-9]. Information theory describes the meaning of *event* in the same way as does the theory of probability:

i) event is appearance of specific element in specific array of elements;
ii) occurrence of an indicated word or sign in a specific message or a specific place of a message;
iii) any of different results of an experiment.

The purpose of a connection between two objects belonging to the same system is the transfer of information from source to its user. The measurement of information may be described as follows. The amount of transferred data depends on the surprise factor (the probability of receiving a message) the lesser the probability, the more information it transfers. The amount of information I_j transferred by the j-th message sent from digital source of information is described as follows, where P_j is the probability of the j-th message. The unit of measure of information is bit.

$$I_j = \log_2 \left(\frac{1}{P_j} \right) \quad (3)$$

Since the probability of different messages is different, there is a difference in transferred data. It is more convenient to call the digital source of information as an *average amount of information* transferred by a single message. By using the definition of an average and the expression of amount of information on a single message, we can write down an expression of average amount of information:

$$H = \sum_{j=1}^{m} P_j I_j = \sum_{j=1}^{m} P_j \log_2 \left(\frac{1}{P_j} \right) \quad (4)$$

where m is the number of possible messages. The average amount of information on a single message is called as an entropy H of source of information.

Amount of information. The amount of information is a measurement of occurrence of an event when the probability is known. It is equal to a logarithm of a unit that is inversely proportional to the probability:

$$I(x) = \log \frac{1}{p(x)} = -\log\ p(x) \quad (5)$$

where $p(x)$ is the probability of an event x. If probability of every event is equal, then the amount of information is equal to the amount of solutions of this set.

General amount of information. The general amount of information is a measure of appearance of two events, x and y. It is equal to a logarithm of a unit inversely proportional to the probability $p(x, y)$ of occurrence of both events simultaneously:

$$I(x, y) = \log \frac{1}{p(x, y)} \quad (6)$$

Relative amount of information. Relative amount of information $I(x\ |y)$ - is a measurement of information about the appearance of an event x when another event y occurred. It is equal to a logarithm of a unit that is inversely proportional to relative probability of an event x:

$$I(x\ |y) = \log \frac{1}{p(x\ |\mathrm{y})} \quad (7)$$

Relative amount of information is equal to the difference between general amount of information of two events and an amount of information of second event.

$$I(x\ |y) = I(x, y) - I(y) \quad (8)$$

Shannon entropy. Shannon entropy H (see Refs. [3-5]) describes the unpredictability of information content. For variable X:

$$H(X) = -\sum P(x) \cdot \log_2(P(x)) \quad (9)$$

where $P(x)$ is the probability that variable X occupies the state x. In case if P=0 (no events)

$$\lim_{P \to 0} (P \cdot \log_2 P(x)) = 0 \quad (10)$$

Shannon entropy allows measurement of the minimal amount of bits required to decode a sequence of symbols based on the frequency of symbols.

Let us assume that probability is the same for every state:

$$P_i = \frac{1}{N} \quad (11)$$

then

$$H(X) = \log_2(N) \quad (12)$$

Entropy is equal to 1 only if the number of all possible states of system is equal to 2.

Rényi entropy, Hartley entropy. Rényi entropy (named in honour of Alfred Rényi) is more general form of Shannon entropy in information theory [10] and it belongs to a branch of functions that evaluate variety, uncertainty and coincidence. Rényi entropy is known as a indexed power function (where index $\alpha \geq 0$)

$$H_\alpha(X) = \frac{1}{1-\alpha} \log_2 \left(\sum_{i=1}^{n} p_i^\alpha \right) \quad (13)$$

where p_i is probability of x_i from a set $\{x_1, x_2, ..., x_n\}$. If all probabilities are equal, Rényi entropy is equal to:

$$H_\alpha(X) = \log_2 n \quad (14)$$

Otherwise entropy is defined as slowly descending functions of α. When $\alpha = 0$, there is a singular point:

$$H_0(X) = \log_2 n = \log_2 |X| \quad (15)$$

where $H_0(X)$ - is called as Hartley entropy of variable X. This is the case when probabilities of receiving a signal are equal. When $\alpha \to 1$, $H_\alpha(X)$ converges to:

$$H_1(X) = \sum_{i=1}^{N} p_i \log_2(p_i) \quad (16)$$

that is Shannon entropy. Rényi entropies are applied in ecology and statistics as a index of diversity.

Information entropy. Entropy in information theory is a value indicating the information gained from the result of an experiment. Information entropy for a system with a finite amount r of states ξ=$\{C_1...C_n\}$ is expressed by equation:

$$H(\xi) = -\sum_{i=1}^{r} p_i \log_2 (p_i) \quad (17)$$

$H(\xi)$ is called as information entropy, where p_i is the probability of i-th state, r - number of states. Information entropy has these features as following.

1. H=0 if and only if all probabilities P_i except one equal zero.
2. For a given number of results n, H is max and equals to $\log_e(n)$, when all probabilities are equal. This is the most uncertain situation.
3. Information entropy is additive: overall entropy of two independent experiments is equal to the sum of separate entropies of these experiments.

Entropy of random results of an experiment that consists of possible results is expressed as follows:

$$H = \log_e(N) \quad (18)$$

Lets assume that the set $X = \{x_1, ..., x_n\}$ is a set of events x_i (i=1,...,n). Expressions $I(x_i)$ are amounts of information of events x_i, $p(x_i)$ as a probabilities of appearance of these events. Also:

$$\sum_{i=1}^{n} p(x_i) = 1 \quad (19)$$

Entropy is the average of information bits that are incompatible between themselves and constituting a full system of events.

$$H(X) = \sum_{i=1}^{n} p(x_i)\ I(x_i) = \sum_{i=1}^{n} p(x_i)\ \log \frac{1}{p(x_i)} \quad (20)$$

When there are two mutually exclusive systems of finite sets consisting of mutually exclusive elements and an event occurs in one of those systems, it is an average value of conditional information values, also known as relative entropy:

Lets assume $X = \{x_1,...,x_n\}$ as a set of events x_i (i=1,...,n). Lets assume $Y = \{y_1,...,y_m\}$ as a set of events y_j (j=1,...,m). Expression $I(x_i|y_j)$ is a conditional amount of information of x_i (if y_j is fulfilled), and if $p(x_i, y_j)$ is overall probability of x_i and y_j.

$$H(X \mid Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i, y_j)\ I(x_i|y_j) \quad (21)$$

Average entropy of a sign:

$$H' = \lim_{m\to\infty} \left(\frac{H_m}{m} \right) \quad (22)$$

there is an overall entropy of m-signs. This limit may be void if signs differ. Average entropy of a sign may be measured by shannons per one sign (1 shannon - unit of information named in honor of Shannon).

***S*-function and *R*-function.** In order to calculate the proportion between order and chaos, we implement the so-called *S-function*:

$$S = \frac{H}{H_{\max} - H} \quad (23)$$

where H is Shannon entropy, H_{max} is the maximum entropy of a system. When probability of every condition is equal (marginal chaos) - it is a continuous function and its values vary from 0 to infinity (see Fig. 1), although this function is not limited. Then we introduce another function with a limit and name it as an *redundancy* or *R*-function:

$$R = \frac{H_{\max} - H}{H_{\max}} = 1 - \frac{H}{H_{\max}} \quad (24)$$

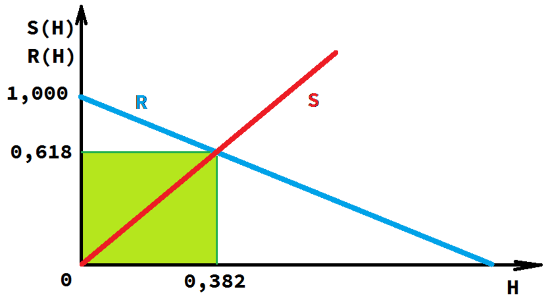

Fig. 1. *S* and *R* functions. Adapted according to Ref. [11].

It is easy that this R-function, unlike an S-function varies from 0 to 1 in an interval $0 \div H_{max}$. We may assume, that when these functions cross each other, system is in a harmonic (balanced) state. After solving the equation, we get the result:

$$H = 0.382 * H_{\max} \tag{25}$$

This expression is called a *golden crossection*. According to Kolkov [11] it is an universal unit of chaos and order. In other words, there has to be 0.382 of chaos and 0.618 of order in total to maintain a harmonic state in a system. System in such state is stable enough, while on the other hand, the level of scatter and uncertainty is high enough to allow changes or further stable evolution.

Shannon and Kolmogorov established that such examples may be found: the limit entropy of any language reaches number of 1.91 bit/character. Furthermore, if we divide the limit value of entropy by maximum, we get the number 0.382 that we have already seen. There were researches in music, poetry and arts, aswell. The relative entropy of top art works is close to the golden crossection.

Antientropy. Antientropy is the cause of appearance of information as well as it's unit of measurement or an expression of organization (grading) of a system. According to the law of Nernst, entropy by itself always increases and may never disappear.

Shannon entropy is compared to a phenomenon called *negentropy*. In the pioneering work [12] Leon Brillouin described the principle of information negentropy saying that obtaining information of micro-aggregate states of a system leads to a decrease in entropy:

i) work has to be done to obtain the information;
ii) deletion leads to increase in thermodynamic entropy.

It complies with the 2nd law of thermodynamics, because, according to Brillouin, reduction in thermodynamic entropy on a local system, causes an increase in entropy elsewhere. Negentropy is a contradictory conception, because effective value of Carnot cycle may be higher than 1. Negentropy is a measurement of both - information and organization of a system.

There is a lot of information considering the informational organization because there are a lot of different entropy forms. Therefore, our focus is placed on the aspects of metrical organization in order to recognise the favourable geometrical properties for development of an observed structure. Following section is devoted for metrical organization

5. Spatial Organization

It is known that some spatial features are imperative for a population to function and to survive in spaces of finite extent [13]. Structures exhibiting such properties have been called as *spatially organized*.

Recognition problem. Populations and other similar structures function remaining under constant disturbances of various factors. Some of them (like scarcity of resources or lack of space) distinguish in growing trends. Namely such disturbances are putting inevitably the strain on population dynamics. A critical situation (crisis) frequently leads to the extinction of a population. However some of them manage to survive learning to exploit more effectively habitats they occupy (specialization). Finally, just very small part of growing populations contrives (sometimes in virtue of lucky mutation) to function and survive outside habitats they upstart. It was deduced [13] that a population to overcome repeating crises should meet the inequality:

$$M(f(x_1, x_2, ..., x_k)) \leq f(M(x_1), (x_2), ..., (x_k)); \tag{26}$$

where $f(...)$ – function describing a spatial unit;
$(x_1, ... , x_k)$ – (geo)metrical features used to describe a unit;
$M(...)$ – symbol of a mean value.

Ineq.(26) is a case of Jensen's inequality [14]. A common structure does not satisfy it (because of the function $f(x_i)$ which as the rule is of convex type). Hence, a structure to survive must change itself insomuch to change this type of interrelationship $f(x_i)$. It was obtained by modeling [15-16] that the negative correlations among some spatial features $(x_1, ... , x_k)$ are able to resolve this contradiction (note that a *functional* feature causes the *spatial* transformation). Expanding structure may not survive ignoring this term. Accordingly to Ineq.(26) can be raised to the status of distinctive sign (or criterion) of spatial organization.

Measuring the organization. The criterion to find the certain structure of population (expressed using Ineq.(26)) often differs in values of ratio r_i:

$$r_i = \frac{M(f(x_1, x_2, ..., x_k))}{f(M(x_1), (x_2), ..., (x_k))} \tag{27}$$

(meaning of variables as in Ineq.(26). It is reasonable to expect (see Ref. [1]) that the smaller is this ratio, the higher is the organization of a given structure. Consequently in order to assess the spatial organization degree M of a structure, the measure can be used [1]:

$$M = 1 - \left[\frac{M(f(x_1, x_2, ..., x_k))}{f(M(x_1), M(x_2), ..., M(x_k))}\right]; \tag{28}$$

(meaning of variables as in Ineq.(26).

Only positive values of this measure from the range [0÷1] are pointing the organization. While the negative values show the structure states, when some part of structure is forced to leave the space suitable for its existence. In this sense the measure M is indicating the capability of a structure to compress itself, sustaining (even increasing) the diversity achieved before the later spatial crisis.

Discussion

Each of measures can be revised by condition associated with the different features (functional, dynamical, etc.). Again, the function $f(x_i)$ is able to represent a spatial unit of any compound structure (population, community, etc.) There is no logical contradiction against such interpretation. Concerning this freedom the questions arise: which of these *virtual* measures would be better revealing the essence of organization? Which of them would be more preferable for assessment of the spatial organization in particular? Or would be more effective for examination of community structure? What sense would have a composition of both measures (say the expression

$$R = 1 - \left[\frac{H(p(r_i))}{H_{max}}\right], \qquad (29)$$

where $p(r_i)$ are the probabilities of ratio values - Eq.(27). There are no as yet compelling answers to these questions significant for understanding of organization as well required for practice. The rather relevant seem has to be any comparative study of these measures.

Conclusions

1. There is no clear motivation how to select the preferable measure for the assessment of spatial arrangement of compound structure.
2. It is unknown yet any direct relation between the functional advantages of a real compound structure and the organization measure R based on the entropy of spatial features.
3. It is reasonable first to get up a comparative study of different measures using the data of real populations or other constituent structures.

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